UNIFIED MODEL DOCUMENTATION PAPER 21

VERTICAL DIFFUSION

by

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<table>
<thead>
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<th>Author</th>
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1. VERTICAL DIFFUSION

This note describes the equations and finite difference formulation of the vertical diffusion which may be applied in the unified forecast/climate model. The term is applied to control small-scale noise that can occasionally develop in the vertical, especially in the wind field at upper levels in low latitudes. It is currently only used for the global forecast version. Its use for other configurations needs to be assessed. The form used in the forecast model (Bell and Dickinson, 1987) did not conserve momentum. A conservative form for the diffusion of momentum is (following the notation of Cullen Davies and Mawson, UM DOC PAPER 10):

\[
\frac{dv}{dt} - \delta_p (F_v)
\]

\[= \delta_p \left( K(\phi, \hat{\phi}) \delta p v \right) \]

\[
K(\phi, \hat{\phi}) = K \frac{(\cos \phi - \cos \hat{\phi})}{(1 - \cos \phi)}
\]

where presently \(K=10^{-8} (Nm^{-2})^{2} s^{-1}\) and \(\phi = \frac{\pi}{6} (30^o)\), although these may be varied by the user. By setting \(K\) to zero the scheme is disabled. \(K=10^2 (Nm^{-2})^{2} s^{-1}\) and \(\phi = \frac{\pi}{6} (30^o)\) in the operational global version. The effective diffusion tails away from the equator and is set to zero for \(\phi > \hat{\phi}\). The diffusion may be applied between any two levels (not necessarily the bottom and top levels of the model - currently it is applied between levels 8 and 14 for the operational global model); to maintain mass weighted conservation of momentum the fluxes at the upper layer boundary of the top level used and the lower layer boundary of the bottom level used are set to zero, thus

\[
(\Delta p)^{n+1} \frac{dv}{dt} \text{top} = -K(\phi, \hat{\phi}) \left( V_{\text{top}} - V_{\text{top-1}} \right) / (\Delta p)^{n+1} \text{top-}\frac{1}{2}
\]

\[
(\Delta p)^{n+1} \frac{dv}{dt} \text{bot} = K(\phi, \hat{\phi}) \left( V_{\text{bot}+1} - V_{\text{bot}} \right) / (\Delta p)^{n+1} \text{bot}+\frac{1}{2}
\]

where the superscript refers to the time level, i.e. the pressure has already been updated by the adjustment step.
Diagnostics of the vertical fluxes of momentum \( \langle F_v \rangle \) are available.

**VERTICAL DIFFUSION (P21) PROGRAM STRUCTURE**

**Calling structure**

ATM_PHYS

- VDIF_CTL
  - P_TO_UV
  - VERT_DIF

**VDIF_CTL Structure**

1. **Initialisation**
   - Set constants for rows, row-lengths etc.

1.1 **Calculate**

\[
K(\phi, \phi') = K \frac{(\cos \phi - \cos \phi') \Delta t}{(1 - \cos \phi')}
\]

for all points. If \( K(\phi, \phi') < 0 \), do not apply diffusion to these points. Set an index where \( K(\phi, \phi') > 0 \). Test whether any points need to be done, if not return with IRET=1.

1.2 **Interpolate** \( \phi^* \) to uv-grid

2. **Gather** points with positive \( K(\phi, \phi') \)

3. **Call** VERT_DIF and update winds
VERT_DIF Structure

1. Calculate

\[ F_{v, k+\frac{1}{2}} = K(\phi \cdot \phi) \left( V_{k+\frac{1}{2}} - U_k \right) / \Delta p_{k+\frac{1}{2}} \]

and update

\[ V_k = \left( F_{v, k+\frac{1}{2}} - F_{v, k-\frac{1}{2}} \right) / \Delta p_k \]

with \( F_v \) set to zero at lower and upper boundaries respectively of lowest and highest levels to be updated.