

UNIFIED MODEL DOCUMENTATION PAPER

No. 26

LARGE-SCALE PRECIPITATION

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Version 5

1 October 1998

Model version 4.5

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P26 Large-scale precipitation

Introduction

Models which do not include prognostic cloud water content variables precipitate all condensed water as soon as it forms. The unified NWP/climate model carries liquid and frozen cloud water variables and thus needs a parametrization of the changes in these due to precipitation processes.

The calculations are done layer by layer looping down from the top model layer with water content variables, layer number Q_LEVELS (called the top "wet" layer). For each layer k there is a liquid (L) and a frozen (F) precipitation rate at the upper interface, $P_{k-1/2}^{(L)}$ and $P_{k-1/2}^{(F)}$ respectively, which are modified within the layer to give the rates at the lower interface, $P_{k-1/2}^{(L)}$ and $P_{k-1/2}^{(F)}$. $P_{1/2}^{(L)}$ and $P_{1/2}^{(F)}$ are the rainfall and snowfall rates respectively at the surface; these are required for the canopy, surface and subsurface hydrology component, P25. P^L and P^F are set to zero at the upper interface of the top wet layer. The precipitation rates are calculated and output as mass fluxes in units of $\text{kgm}^{-2}\text{s}^{-1}$.

The first processes dealt with for a given layer are the evaporation of rainfall and sublimation of frozen precipitation as described in (a) below. Secondly the calculation of latent heating/cooling due to any freezing/melting of the cloud water and precipitation is described in (b). Lastly, the model calculates changes due to the exchanges between frozen or liquid cloud water contents and the corresponding precipitation rates, described in (c).

In the following sections a superscript n indicates the value of a quantity on entry to P26 and superscript $n+1$ its value on exit. A tilde (\sim) indicates an intermediate value after evaporation and sublimation of precipitation and a hat ($\hat{}$) an updated tilde value after freezing or melting of cloud water and precipitation.

(a) Evaporation and sublimation of precipitation (subroutine LSP_EVAP)

The rates of change of specific humidity due to evaporation (EV) or sublimation (SB) of precipitation are parametrized as:

$$(\dot{q}_k)_{PEV} = c_{EV} (q_{SAT}(T_k, p_k) - q_k) \quad (\text{P26.1a})$$

$$(\dot{q}_k)_{PSB} = c_{SB} (q_{SAT}(T_k, p_k) - q_k) \quad (\text{P26.2a})$$

where c_{EV} and c_{SB} are bulk evaporation rate coefficients (see below). The time discretisation of these incorporates an implicit treatment:

$$(\dot{q}_k)_{PEV} = (\tilde{q}_k - q_k^n) / \delta t = c_{EV}^n (q_{SAT}(\tilde{T}_k, p_k^n) - \tilde{q}_k) \quad (\text{P26.1b})$$

$$(\dot{q}_k)_{PSB} = (\tilde{q}_k - q_k^n) / \delta t = c_{SB}^n (q_{SAT}(\tilde{T}_k, p_k^n) - \tilde{q}_k) \quad (\text{P26.2b})$$

The corresponding temperature rates of change are:

$$(\dot{T}_k)_{PEV} = - \left(\frac{L_C}{c_p} \right) (\dot{q}_k)_{PEV} \quad (\text{P26.3})$$

$$(\dot{T}_k)_{PSB} = - \left(\frac{L_C + L_F}{c_p} \right) (\dot{q}_k)_{PSB} \quad (\text{P26.4})$$

Combining (P26.1b and 3) and (P26.2b and 4) and expanding q_{SAT} to first order about T_k^n gives

$$(\dot{q}_k)_{PEV} = (c_{EV}^n / b_L) (q_{SAT}(T_k^n, p_k) - q_k) \quad (\text{P26.1c})$$

$$(\dot{q}_k)_{PSB} = (c_{SB}^n / b_L) (q_{SAT}(T_k^n, p_k) - q_k) \quad (\text{P26.2c})$$

where

$$b_L = 1 + c_{EV}^n \delta t (1 + L_C \alpha_L / c_p) \quad (\text{P26.101})$$

$$b_F = 1 + c_{SB}^n \delta t (1 + (L_C + L_F) \alpha_F / c_p) \quad (\text{P26.102})$$

$$\alpha_L = \frac{\varepsilon L_C q_{SAT}(T_k^n, p_k^n)}{R(T_k^n)^2} \quad (\text{P26.103})$$

$$\alpha_F = \frac{\varepsilon(L_C + L_F) q_{SAT}(T_k^n, p_k^n)}{R(T_k^n)^2} \quad (\text{P26.104})$$

The implicit treatment ensures that the layer doesn't supersaturate if $c_{EV} \delta t$ or $c_{SB} \delta t$ are large.

No more than the total amount of precipitation should be allowed to evaporate. Also a minimum value of zero for the subsaturation $q_{SAT}(T,p) - q$ should be used; this quantity may be less than zero on entry to the subroutine because the temperature may have changed since the large-scale cloud scheme was called. With these considerations the formulae used in the code are:

$$(\dot{q}_k)_{PEV} = \min \left(\frac{P_{k-1/2}^{(L)}}{(\rho \Delta z)_k}, \frac{c_{EV}^n}{b_L} \max(0, [q_{SAT}(T_k^n, p_k^n) - q_k^n]) \right) \quad (\text{P26.1d})$$

$$(\dot{q}_k)_{PSB} = \min \left(\frac{P_{k-1/2}^{(F)}}{(\rho \Delta z)_k}, \frac{c_{SB}^n}{b_F} \max(0, [q_{SAT}(T_k^n, p_k^n) - q_k^n]) \right) \quad (\text{P26.2d})$$

where $(\rho \Delta z)_k = -\Delta p_k / g$ and $\Delta p_k = \Delta A_k + p_k^n \Delta B_k$.

The temperature and humidity are updated using

$$\tilde{T}_k = T_k^n + \delta t \left((\dot{T}_k)_{PEV} + (\dot{T}_k)_{PSB} \right) \quad (\text{P26.5})$$

$$\tilde{q}_k = q_k^n + \delta t \left((\dot{q}_k)_{PEV} + (\dot{q}_k)_{PSB} \right) \quad (\text{P26.6})$$

The precipitation rates are reduced to balance the water budget as follows:

$$\tilde{P}^{(L)} = P_{k-1/2}^{(L)} - (\rho \Delta z)_k (\dot{q}_k)_{PEV} \quad (\text{P26.7})$$

$$\tilde{P}^{(L)} = P_{k-1/2}^{(F)} - (\rho \Delta z)_k (\dot{q}_k)_{PSB} \quad (\text{P26.8})$$

The derivation of expressions for the bulk evaporation rate coefficients C_{EV} and C_{SB} are described by Gregory (1995). Note that **version 1A** of the precipitation scheme uses a much earlier version of evaporation and should not be used. **Version 2B/C** uses the evaporation formulation below, but coefficients are calculated assuming a constant pressure of 1000mb. Also in deriving C_{EV} the raindrops are assumed to follow the Marshall-Palmer drop size distribution. **Version 2D** contains an updated evaporation scheme in which the evaporation rate coefficients include the correct pressure dependence and C_{EV} is estimated assuming a gamma distribution of order unity to describe the drop size distribution.

Hence in **version 2B/C** C_{EV} and C_{SB} are given by;

$$C_{EV} = A_{EV}(T) (75.67(R\rho^{0.5})^{0.42} + 605.17\rho^{0.55}R^{0.60}) \quad (P26.9)$$

$$C_{SB} = A_{SB}(T) (1765.55(S\rho^{0.5})^{0.55} + 34784.06\rho^{0.63}S^{0.76})$$

while in **version 2D**,

$$C_{EV} = A_{EV}(T) (10^5/\rho) (122.68(R\rho^{0.5})^{0.52} + 919.25\rho^{0.55}R^{0.67}) \quad (P26.10)$$

$$C_{SB} = A_{SB}(T) (10^5/\rho) (1765.55(S\rho^{0.5})^{0.55} + 34784.06\rho^{0.63}S^{0.76})$$

where R and S are precipitation rates (in $\text{kgm}^{-2}\text{s}^{-1}$) $A_{EV}(T)$ and $A_{SB}(T)$ are quadratics in temperature;

$$A_{EV} = 2.008 \times 10^{-9}T^2 - 1.385 \times 10^{-6}T + 2.424 \times 10^{-4} \quad (P26.11)$$

$$A_{SB} = -5.2 \times 10^{-9}T^2 + 2.5332 \times 10^{-6}T - 2.9111 \times 10^{-4} \quad (P26.12)$$

The quadratic A_{SB} has a root at 243.58K and so for temperatures lower than this the value at 243.58K is used. Derivation of these quadratics is explained in Gregory (1995). To a large extent, they account for the change in the diffusivity of water with temperature.

c_{EV}^n and c_{SB}^n in (P26.1d) and (P26.2d) are values calculated from (P26.9) and (P26.10)

using T_k^n , $P_{k-1/2}^{(L)}$ and $P_{k-1/2}^{(F)}$.

(b) Freezing and melting of cloud water and precipitation (subroutine LSP_FRMT)

Since the frozen and liquid cloud water were calculated in the large-scale cloud subroutine, other physical processes may have changed the temperature. This implies that the cloud water may be in an inappropriate state. If so, it is desirable (but not essential for energetic consistency) to melt or freeze the cloud water before it is converted to liquid or frozen precipitation. This, along with the latent heating, is done according to the algorithm:

if $\tilde{T} - T_M \leq 0$:

$$q_c^{FREEZE} = \min(q_c^{(L)n}, c_p(T_M - \tilde{T})/L_F) \quad (P26.13)$$

$$\hat{T} = \tilde{T} + L_F q_c^{FREEZE} / c_p \quad (P26.14)$$

$$\hat{q}_c^{(L)} = q_c^{(L)n} - q_c^{FREEZE} \quad (P26.15)$$

$$\hat{q}_c^{(F)} = q_c^{(F)n} - q_c^{FREEZE} \quad (P26.16)$$

otherwise:

$$q_c^{MELT} = \min(q_c^{(F)n}, c_p(\tilde{T} - T_M)/L_F) \quad (P26.17)$$

$$\hat{T} = \tilde{T} - L_F q_C^{MELT} / c_p \quad (P26.18)$$

$$\hat{q}_C^{(L)} = q_C^{(L)n} - q_c^{MELT} \quad (P26.19)$$

$$\hat{q}_C^{(F)} = q_C^{(F)n} - q_c^{MELT} \quad (P26.20)$$

Sometimes there may be more liquid/frozen cloud water than can be frozen/melted. This is taken into account by the "min" function in (P26.13) and (P26.17). If the cloud water is incompletely frozen or melted, a mixture of liquid and frozen cloud water at temperature T_M is predicted. Note that the change of state of cloud water conserves the cloud-conserved thermodynamic variable T_L , as it should:

$$\begin{aligned} \hat{T}_L &= \hat{T} - (L_C / c_p) \hat{q}_c^{(L)} - ((L_C + L_F) / c_p) \hat{q}_c^{(F)} \\ &= \hat{T} - (L_C / c_p) q_c^{(L)n} - ((L_C + L_F) / c_p) q_c^{(F)n} = \tilde{T}_L \end{aligned}$$

After the cloud water change of state, the melting or freezing of precipitation and associated latent heating is calculated by assuming that this takes place instantaneously in the model layer. As with cloud water, sometimes there may be more liquid/frozen precipitation in a layer than can be frozen/melted: if so a mixture of liquid and frozen precipitation ("sleet") is output from the subroutine. The algorithm used (for layer k) is:

if $\hat{T}_k - T_M \leq 0$:

$$P^{FREEZE} = \min \left[\tilde{\rho}^{(L)}, \frac{c_p (T_M - \hat{T}_k) (\rho \Delta z)_k}{L_F \delta t} \right] \quad (P26.21)$$

$$(\dot{T})_{FMP} = \frac{L_F}{c_p} \frac{\rho^{FREEZE}}{(\rho \Delta z)_k}; T_k^{n-1} = \hat{T}_k + \delta t (\dot{T})_{FMP} \quad (P26.22)$$

$$\hat{\rho}^{(L)} = \tilde{\rho}^{(L)} - P^{(FREEZE)} \quad (P26.23)$$

$$\hat{\rho}^{(F)} = \tilde{\rho}^{(F)} + P^{(FREEZE)} \quad (P26.24)$$

otherwise:

$$P^{(MELT)} = \min \left[\tilde{\rho}^{(F)}, \frac{c_p (\hat{T}_k - T_M) (\rho \Delta z)_k}{L_F \delta t} \right] \quad (P26.25)$$

$$(\dot{T})_{FMP} = - \frac{L_F}{c_p} \frac{\rho^{(MELT)}}{(\rho \Delta z)_k}; T_k^{n-1} = \hat{T}_k + \delta t (\dot{T})_{FMP} \quad (P26.26)$$

$$\hat{\rho}^{(L)} = \tilde{\rho}^{(L)} + P^{(MELT)} \quad (P26.27)$$

$$\hat{\rho}^{(F)} = \tilde{\rho}^{(F)} - P^{(MELT)} \quad (P26.28)$$

(c) The formation of precipitation (subroutine LSP_FORM)

There is in reality an important difference between liquid water and ice clouds, which the precipitation scheme incorporates. In water clouds the available condensed water is spread over many cloud condensation nuclei (CCN) and few droplets are large enough to fall out (Pruppacher and Klett, 1978). The depletion rate of cloud liquid water is equal to the

formation rate of the large droplets which, once formed, fall out as rain very rapidly. On the other hand, very few CCN act as ice nuclei (Heymsfield and Sabin, 1989) and cloud ice is, from the start, spread over fewer, larger particles, all of which quickly develop appreciable fall speeds (Heymsfield, 1977). The rate of change of cloud ice in a layer is, therefore, the divergence across the layer of the flux of these falling ice particles.

The precipitation formation scheme **version 2B/C** has recently undergone changes, to the mixed phase cloud partitioning and ice fallout sections, as well as the inclusion of the large-scale cloud moisture distribution in the rate calculations, which are included as **version 2D**. The basic scheme is now described, followed by an explanation of the **2D** differences.

Version 2B/C

The microphysics of mixed-phase cloud are complex and not fully understood. Interactions between the liquid and frozen components of cloud water and precipitation should probably be parametrized. However, in the current version of the model a much simpler approach is used. *For the purpose of calculating precipitation formation*, (in contrast to the latent heating), all cloud water is assumed to be liquid for temperatures above 0°C and all to be ice below -15°C. In the intermediate temperature range, the proportion of cloud liquid water f_L in a grid-box is assumed to be given by a quadratic spline:

$$f_L = \begin{cases} 1/6((T - T_M + 15)/5)^2 & \text{for } -15 < T - T_M < -5 \\ 1 - 1/3((T - T_M)/5)^2 & \text{for } -5 \leq T - T_M < 0 \end{cases} \quad (\text{P26.29})$$

giving a smooth transition between ice and liquid states for the ensemble of clouds in a grid-box. For individual clouds, in reality, the transition is more abrupt and depends on factors other than local temperature such as cloud depth and cloud-top temperature. All these effects are intended to be represented by (P26.29) for the ensemble. The form of the partitioning curve and the lower bound of -15°C for liquid water do not have a strong basis in observation or theory. The form was chosen for its simplicity and smoothness, and the lower bound was chosen to give realistic cloud amounts.

For the purposes of parametrizing precipitation processes (and also for calculating the radiative properties of cloud (see component P23) the cloud liquid water content q_{CL} and cloud ice content q_{CF} are given in terms of f_L by

$$q_{CL} = f_L q_c, \quad q_{CF} = (1 - f_L) q_c \quad \text{where} \quad q_c = q_c^{(L)} + q_c^{(F)} \quad (\text{P26.30})$$

There is an inconsistency in applying the partition between liquid water and ice given by (P26.30) for precipitation and radiation calculations but, for the thermodynamics, assuming that cloud water and precipitation release or take up latent heat only at 0°C, i.e. in general $q_{CL} \neq q_c^{(L)}$ and $q_{CF} \neq q_c^{(F)}$. This does not unbalance the model's energy or water budgets and so is not too serious. Nevertheless, further development of the large-scale precipitation scheme should aim to eliminate this inconsistency.

The current scheme treats separately the cloud water which is ice for the thermodynamics but liquid for the precipitation calculations: this is q_{CA} given by

$$q_{c\Delta} = q_{cL} - q_c^{(L)} = q_c^{(F)} - q_{cF} \quad (\text{P26.301})$$

$$q_c = q_c^{(L)} + q_{c\Delta} + q_{cF} \quad (\text{P26.302})$$

The three cloud water components represented by (P26.302) are, respectively, the liquid water, the ice-treated-as-liquid for fallout calculations and the ice-treated-as-frozen.

The conversion rate to precipitation of the liquid water, i.e. $q_c^{(L)}$ and $q_{c\Delta}$, is parametrized as:

$$(\dot{q}_{cX})_{PPN} = -C \left\{ c_T \left[1 - \exp \left[- \left(\frac{\rho q_c / C}{c_w} \right)^2 \right] \right] + \frac{P}{c_A} \right\} \frac{Q_{cX}}{C} \quad (\text{P26.31})$$

where q_{cX} is either $q_c^{(L)}$ or $q_{c\Delta}$.

The values used for the parameters in this expression are:

$$c_T = 10^{-1} \text{ s}^{-1}, \quad c_w = 0.8 \times 10^{-3} \text{ kg m}^{-3} \text{ (for land points), } 0.2 \times 10^{-3} \text{ kg m}^{-3} \text{ (for sea points)}$$

and $c_A = 1 \text{ kg m}^{-2}$. P is the mass flux of precipitation (liquid and frozen) entering the layer from above after allowing for any evaporation or sublimation. The density, ρ is found from equation of state, $\rho = p/(RT_v)$, where p is pressure and T_v is the virtual temperature.

Equation (P26.31) is based on the parametrization proposed by Sundqvist (1978 and 1981) but with an additional accretion term P/c_A which increases the conversion rate when precipitation falls into a layer (Golding, 1986). The leading factor C is present because precipitation only forms in that fraction of the gridbox where there is cloud. Autoconversion of cloud droplets to rain is represented by the term involving c_T . The exponential factor in this term inhibits conversion when the in-cloud condensed water density, $\rho q_c / C$, is small compared with c_w . Because air over oceans has less cloud condensation nuclei, the drops grow larger and precipitate more easily. This implies that oceanic clouds can hold less water and thus that a lower value of c_w is appropriate over sea points.

For ice clouds, if the precipitating ice fall-out speed is v_F , the flux of precipitation leaving a layer due to this cloud ice fallout is $\rho q_{cF} v_F$. This implies, by taking the flux divergence across the layer, a rate of change of cloud ice water given by:

$$(\dot{q}_{cF})_{PPN} = (P_* - \rho q_{cF} v_F) / (\rho \Delta Z) \quad (\text{P26.32})$$

P_* is the proportion of the frozen precipitation which enters the layer (after allowing for changes of state) and is assumed to remain there, rather than falling through.

v_F parametrizes in terms of the in-cloud water content, using the formula deduced from observations by Heymsfield (1977):

$$v_F = v_{F1} \left[\frac{\rho q_C / C}{c_F} \right]^{0.17} \quad (\text{P26.33})$$

v_{F1} is set to 1 m s^{-1} and c_F to $1.01086 \times 10^{-3} \text{ kg m}^{-3}$.

The equations (P26.31) and (P26.32) for layer k can be written as

$$(\dot{q}_C^{(L)})_{PPN} = -R_{CL} q_C^{(L)} \quad (\text{P26.34})$$

$$(\dot{q}_{C\Delta})_{PPN} = -R_{CL} q_{C\Delta} \quad (\text{P26.35})$$

$$(\dot{q}_{CF})_{PPN} = -R_{CF} q_{CF} + \frac{(1-f_{PL})(1-f_{P\Delta(k-1)})\hat{P}^{(F)}}{\rho \Delta Z} \quad (\text{P26.36})$$

$f_{P\Delta}$ is the proportion of the frozen precipitation entering the layer which is treated as liquid in precipitation calculations and therefore is assumed to fall straight through the layer, contributing to the precipitation flux at its lower interface. The remaining fraction, $1 - f_{P\Delta}$ is treated as frozen and is assumed to increase q_{CF} in that layer. f_{PL} is the proportion of ice-treated-as-frozen precipitation that is allowed to fall straight through the layer, in the same manner as liquid precipitation. In **version 2B** $f_{PL}=0$, and all ice-treated-as-frozen fallout from layer k is deposited in layer $k-1$, whereas **version 2C** has $f_{PL}=1$, forcing all precipitation directly to the surface (**version 2D** dispenses with this variable). R_{CL} and R_{CF} are given by

$$R_{CL} = C_T \left\{ 1 - \exp \left[- \left(\frac{\rho q_C / C}{c_W} \right)^2 \right] \right\} + \frac{P}{c_A} \quad (\text{P26.37})$$

$$R_{CF} = v_F / \Delta Z \quad (\text{P26.38})$$

where ΔZ is calculated as $(\rho \Delta Z) / \rho$.

(P26.34)-(P26.36) are treated with an implicit timestepping scheme, i.e. with $q_C^{(L)}$, $q_{C\Delta}$ and q_{CF} on the right hand sides taken at timelevel $n+1$. This ensures that unphysical negative values are not produced. R_{CL} and R_{CF} are calculated using timelevel n values. The updated liquid and frozen cloud water contents in layer k are then given by

$$q_C^{(L)n-1} = \frac{\hat{q}_C^{(L)}}{(1 + R_{CL} \delta t)} \quad (\text{P26.39})$$

$$q_{C\Delta}^{n-1} = \frac{\hat{q}_{C\Delta}}{(1 + R_{CL} \delta t)} \quad (\text{P26.40})$$

$$q_{CF}^{n-1} = \frac{\hat{q}_{CF}}{(1 + R_{CF} \delta t)} + \frac{\delta t (1-f_{PL})(1-f_{P\Delta(k-1)})\hat{P}^{(F)}}{(\rho \Delta Z)_k (1 + R_{CF} \delta t)} \quad (\text{P26.41})$$

$$q_C^{(F)n-1} = q_{C\Delta}^{n-1} + q_{CF}^{n-1} \quad (\text{P26.42})$$

The precipitation fluxes at the lower interface of the layer are given by

$$P_{k-1/2}^{(L)} = (\rho\Delta z)_k R_{CL} q_C^{(L)n-1} + \hat{P}^{(L)} \quad (\text{P26.43})$$

$$P_{k-1/2}^{(\Delta)} = (\rho\Delta z)_k R_{CL} q_{C\Delta}^{n-1} + f_{P\Delta(k-1)} \hat{P}^{(F)} \quad (\text{P26.44})$$

$$P_{k-1/2}^{(F)} = (\rho\Delta z)_k R_{CF} q_{CF}^{n-1} + f_{PL}(1-f_{P\Delta(k-1)}) \hat{P}^{(F)} + P_{k-1/2}^{(\Delta)} \quad (\text{P26.45})$$

The precipitation treated as liquid, $\hat{P}^{(L)}$ and $f_{P\Delta(k-1)} \hat{P}^{(F)}$, is assumed to fall straight through the layer and contributes to the flux at its lower interface.

It can easily be shown that liquid, frozen-treated-as-liquid and total frozen water are conserved by (P26.39)-(P26.45):

$$\frac{(q_C^{(L)n-1} - \hat{q}_C^{(L)})}{\delta t} = \frac{(\hat{P}^{(L)} - P_{k-1/2}^{(L)})}{(\rho\Delta z)_k} \quad (\text{P26.46})$$

$$\frac{(q_C^{n-1} - \hat{q}_{C\Delta}^{(L)})}{\delta t} = \frac{(f_{P\Delta(k-1)} \hat{P}^{(F)} - P_{k-1/2}^{(\Delta)})}{(\rho\Delta z)_k} \quad (\text{P26.47})$$

$$\frac{(q_C^{(F)n-1} - \hat{q}_C^{(F)})}{\delta t} = \frac{(\hat{P}^{(F)} - P_{k-1/2}^{(F)})}{(\rho\Delta z)_k} \quad (\text{P26.48})$$

Lastly the fraction $f_{P\Delta}$ is calculated for layer k ,

$$f_{P\Delta(k)} = P_{k-1/2}^{(\Delta)} / P_{k-1/2}^{(F)} \quad (\text{P26.49})$$

($f_{P\Delta(k)}$ is set to zero if $P_{k-1/2}^{(F)}$ is zero). $f_{P\Delta(k)}$ is stored for use in the layer $k-1$ calculations.

Version 2D

a) Mixed phase cloud partition function.

Although, originally, the formulation of the temperature dependent f_L , used to estimate the fraction of cloud condensate which is water, was purely empirical, observations provided by the Met. Research Flight suggest a narrower temperature range of 0°C to -9°C. The cloud water is assumed to be all liquid for temperatures above 0°C and all ice below -9°C. In the intermediate temperature range, the proportion of cloud liquid water f_L in a grid-box varies linearly:

$$f_L = \frac{T}{9} - 29.35 \quad \text{for } -9 \leq T - T_M \leq 0K \quad (\text{P26.50})$$

giving a smooth transition between ice and liquid states for a grid-box cloud ensemble.

The proposed move to a combined cloud and precipitation microphysics scheme will allow direct prediction of cloud ice-water contents and render this f_L parametrization obsolete. **WARNING:** altering the value of T_M may require a different parametrization to be adopted.

b) Changes in the conversion rate calculations.

Using the new mixed-phase cloud partition function, the grid-box mean cloud water components of equation (P26.302) may again be calculated. The mean cloud fraction and cloud water contents are calculated within the large-scale cloud scheme (UM Documentation Paper 29), based upon a statistical description of the total water distribution within a grid box. As pointed out by Jonas (1993 - private communication), including the same statistical moisture distribution into the precipitation formation equations will alter the equilibrium between cloud cover and precipitation in favour of lower cloud amounts, by allowing increased rainfall from cloud patches where the local moisture content exceeds the mean. The large-scale cloud scheme uses a gridbox mean supersaturation, Q_C (P292.3), its local deviation, s (P292.4), and a prescribed probability distribution function, $G(s)$, to calculate the cloud fraction

$$C = \int_{-Q_c}^{\infty} G(s) ds \quad (\text{P292.9})$$

and grid-box mean cloud water content

$$q_c = \int_{-Q_c}^{\infty} (s + Q_c) G(s) ds \quad (\text{P292.10})$$

The function $G(s)$ is assumed to be

$$G(s) = \begin{cases} (b_s + s)/b_s^2 & \text{for } -b_s < s \leq 0 \\ (b_s - s)/b_s^2 & \text{for } 0 < s < b_s \\ 0 & \text{else} \end{cases}$$

where b_s (P292.14) is proportional to $1 - RH_{\text{Critical}}$.

By importing the cloud scheme values of Q_C and b_s , the precipitation formation rates (P26.31-32) may be reformulated along the same lines as the cloud fraction and cloud water content equations above:

$$(\dot{q}_{CL})_{PPN} = -\frac{q_{CL}P}{c_A} - f_L c_T \int_{-Q_c}^{b_s} \left(1 - \exp \left[-\left(\frac{\rho(s + Q_c)}{c_W} \right)^2 \right] \right) (s + Q_c) G(s) ds \quad (\text{P26.51})$$

$$(\dot{q}_{CF})_{PPN} = -\frac{1}{\rho \Delta Z} \left(P_* - (1 - f_L) \rho v_{F1} \int_{-Q_c}^{b_s} \left(\frac{\rho(s + Q_c)}{c_F} \right)^{0.17} (s + Q_c) G(s) ds \right) \quad (\text{P26.52})$$

where the various parameters are set as before.

In accordance with (P26.34), the liquid water content equation gives:

$$R_{CL} = \frac{P}{c_A} + \begin{cases} 0 & \dots -b_s \geq Q_c \\ C_q (NERF[\sigma_-]) & \dots -b_s < Q_c \leq 0 \\ C_q (NERF[\sigma_-] - 2NERF[\rho Q_c/c_W]) & \dots 0 < Q_c \leq b_s \\ C_q (NERF[\sigma_-] - 2NERF[\rho Q_c/c_W] + NERF[\sigma_-]) & \dots b_s < Q_c \end{cases} \quad (\text{P26.53})$$

where $\sigma_- = \rho(Q_c + b_s)/c_W$: $\sigma_- = \rho(Q_c - b_s)/c_W$: $NERF[x] = (x^3/3) - x + (\sqrt{\pi}/2)ERF[x]$,

$ERF[x]$ is the numerically evaluated error function and $C_q = \frac{c_T}{2q_c} \left(\frac{c_W}{\rho} \right)^3 \left(\frac{1}{b_s} \right)^2$.

(P26.34) and (P26.35) are treated with an implicit timestepping scheme, i.e. with $q_C^{(l)}$, $q_{c\Delta}$ on the right hand sides taken at timelevel $n+1$ to prevent production of unphysical negative values. R_{CL} is calculated using timelevel n values. The updated liquid and frozen cloud water contents in layer k are then given by equations (P26.39) and (P26.40) as in the **version 2B/C** scheme.

However, the frozen cloud water fallout scheme has undergone modifications in addition to the inclusion of a moisture distribution. **Version 2D** allows ice to fall through more than one model layer under the condition:

$$\hat{v}_F^{k-1} \delta t > \Delta Z_k \quad (\text{P26.54})$$

where \hat{v}_F^{k-1} is the speed of ice falling into the layer, δt is the timestep and ΔZ_k is the layer thickness.

When (P26.54) is not satisfied, ice falling into layer k will not cross into layer $k-1$ within a timestep and the divergent flux solution (P26.52) is valid. This is the **short-timestep** case (although, strictly speaking, the timestep is fixed in a model run and the inequality requires large layer thicknesses or small speeds for ice falling into the layer). From (P26.52):

$$\frac{\rho v_F}{\rho \Delta Z_k} = \frac{v_{F1} \left(\frac{\rho}{c_F} \right)^\alpha}{(2+\alpha)(3+\alpha)\Delta Z_k q_C b_s^2} \begin{cases} 0 & \dots -b_s \geq Q_c \\ (Q_c + b_s)^{3-\alpha} & \dots -b_s < Q_c \leq 0 \\ (Q_c + b_s)^{3-\alpha} - 2Q_c^{3-\alpha} & \dots 0 < Q_c \leq b_s \\ (Q_c + b_s)^{3-\alpha} - 2Q_c^{3-\alpha} + (Q_c - b_s)^{3-\alpha} & \dots b_s < Q_c \end{cases} \quad (\text{P26.55})$$

where $\alpha = 0.17$.

Note that this approach does not produce a strict grid-box mean speed, as would be found from (P26.33). Instead, ρv_F is defined such that multiplication by the mean ice cloud water content q_{CF} will reproduce the correct grid-box mean flux.

For the **short timestep** case, ice falling out of the layer has the layer fallout velocity:

$$v_{F(k)} = v_F \quad (\text{P26.56})$$

But the fallout flux is calculated with the previous implicit timestepping replaced by an explicit solution. This reduces the model's tendency to form large quantities of very thin cirrus, which results from the implicit formulation's constraint that a residual cloud water content be left in the layer. In order to avoid creating unphysical, negative cloud water contents, the fallout flux (R_{CF}^* , now in kg/ m²/ s) is limited:

$$R_{CF}^* = \begin{cases} \hat{q}_{CF} \rho v_F & \text{for } v_F \delta t \leq \Delta Z_k, \\ \hat{q}_{CF} \rho \Delta Z_k / \delta t & \text{otherwise.} \end{cases} \quad (\text{P26.57})$$

and the updated frozen cloud water becomes:

$$q_{CF}^{n-1} = \hat{q}_{CF} + \frac{\delta t ((1 - f_{P\Delta(k-1)}) \hat{P}^{(F)} - R_{CF}^*)}{\rho \Delta Z_k} \quad (\text{P26.58})$$

In the **long timestep** case, ice falling into the layer is assumed to sweep up any frozen cloud water initially present in the layer and carry it into the layer below. Ice is taken to fall

into layer $k-1$ at the faster of the layer $k+1$ and k fallout speeds:

$$v_{F(k)} = \begin{cases} \hat{v}_F^{k-1} & \text{for } \hat{v}_F^{k-1} \geq v_F \\ v_F & \text{else.} \end{cases} \quad (\text{P26.59})$$

The timestep is long enough that all memory of the initial ice cloud water content in layer k is overwritten by the incoming layer $k+1$ ice flux, which equilibrates across layer k . The updated frozen cloud water depends solely upon the flux equilibrium condition:

$$q_{CF}^{n-1} = \frac{(1 - f_{P\Delta(k-1)}) \hat{P}^{(F)}}{\rho \hat{v}_F^{k-1}} \quad (\text{P26.60})$$

The layer k fallout flux is obtained from the discretized moisture conservation equation:

$$\frac{q_{CF}^{n-1} - q_{CF}^n}{\delta t} = \frac{\left((1 - f_{P\Delta(k-1)}) \hat{P}^{(F)} \right) - R_{CF}^*}{(\rho \Delta Z)_k}$$

giving

$$R_{CF}^* = \left((1 - f_{P\Delta(k-1)}) \hat{P}^{(F)} \left(1 - \frac{\Delta Z_k}{\delta t \hat{v}_F^{k-1}} \right) \right) + \frac{\hat{q}_{CF} \rho \Delta Z_k}{\delta t} \quad (\text{P26.61})$$

With q_{CF}^{n-1} defined for both **short and long timestep** conditions, the ice-treated-as-liquid and ice-treated-as-frozen terms may again be combined (P26.42). The precipitation fluxes at the layer's lower interface are given by:

$$P_{k-1/2}^{(L)} = (\rho \Delta Z)_k R_{CL} q_C^{(L)n-1} + \hat{P}^{(L)} \quad (\text{P26.43})$$

$$P_{k-1/2}^{(\Delta)} = (\rho \Delta Z)_k R_{CL} q_{C\Delta}^{n-1} + f_{P\Delta(k-1)} \hat{P}^{(F)} \quad (\text{P26.44})$$

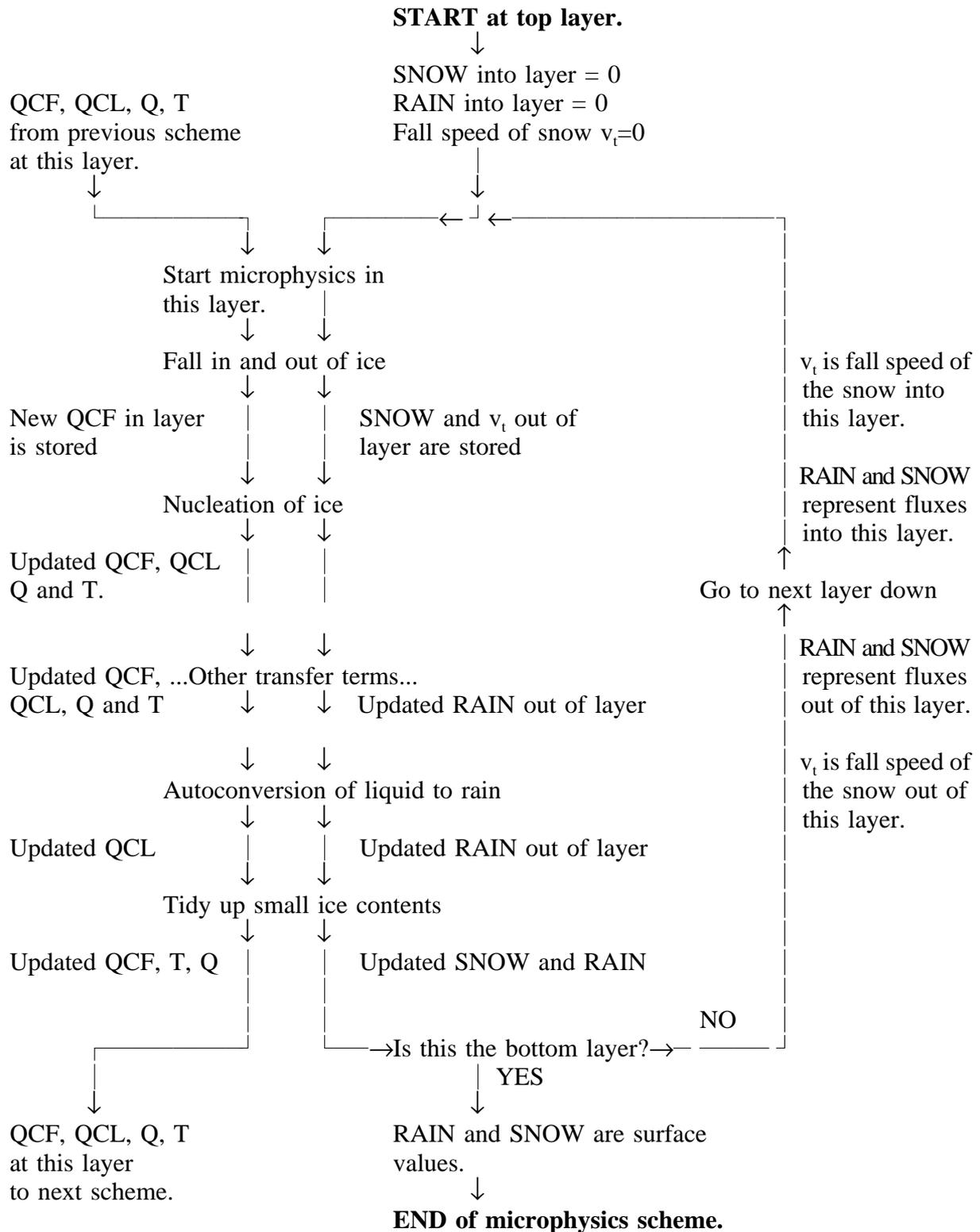
$$P_{k-1/2}^{(F)} = R_{CF}^* + P_{k-1/2}^{(\Delta)} \quad (\text{P26.62})$$

The fraction of snow treated as rain is calculated as before (P26.49).

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APPENDIX: FLOW CHART FOR 3A LARGE SCALE PRECIPITATION SCHEME.



3B AND 3A LARGE SCALE PRECIPITATION SCHEMES.

INTRODUCTION.

This paper documents the 3B large scale precipitation scheme, and explains how the 3A scheme differs. The 3B and 3A large scale precipitation schemes should only be run in conjunction with the 2A or 2B cloud scheme which are documented separately. They should not be run with the 1A cloud scheme. Since the 3B and 3A large scale precipitation schemes are considerably different from version 2D (and previous versions) this paper is complete without reference to the 2D documentation. *Version 3B is the recommended version of the mixed phase precipitation scheme. One should not run with 3A as it contains some out of date formulations. Unless specifically stated, this paper documents the 3B version.*

Overview.

The mixed phase precipitation scheme is a physically based transfer scheme. Its basic variables represent water vapour, liquid water, ice water (in all its forms) and rain. The scheme attempts to quantify the transfer of moisture from one phase to another using equations based upon cloud physics, such as the microphysics of heat and vapour transfer from the surroundings to an ice particle. It avoids as far as possible any parameterizations based upon a tunable number which cannot be quantified by physical observation. There are many processes occurring within a cloud, most of which are incredibly complex due to the huge variety of shapes and sizes of particles which are present. Inevitably, simplifications have to be made. Only the major processes are included in the scheme while some important processes, such as secondary ice nuclei production, are excluded. The properties of ice particles and raindrops have to be parameterized in terms of their diameters. However, despite the simplifications, the mixed phase scheme is the first unified model parameterization scheme which produces a reasonable prediction of cloud phase, since it is based upon a physical prediction of ice content rather than using a diagnostic parameterization.

The full discussion of the scheme is necessarily complicated. The layout of this paper is to firstly describe the structure of the scheme, then to describe the basic parameterizations of the properties of ice particles and raindrops in terms of their diameters. A brief summary of 'sub grid' scale issues follows - any parameterization scheme is supposed to be describing an ensemble of clouds in a grid box whose size can be of order 10 km to order 300km. Then each of the parameterized transfer terms are described in detail. These are complex descriptions and their explanation takes up most of this paper.

General Structure of the Scheme.

Previous large scale precipitation schemes do not include a prognostic treatment of the cloud phase and diagnose this each timestep. In this scheme ice is considered a prognostic variable, but liquid is diagnosed each timestep from a cloud scheme. Rain is diagnosed each timestep. The 3A large scale precipitation scheme converts the input water quantities of vapour, Q, liquid, QCL, and ice, QCF, into new values of vapour, liquid and ice, and diagnoses a rain (and snow) fall out. Q, QCL and QCF all have units of kg kg^{-1} . Scientifically, ice and snow are equivalent. In this documentation ice refers to QCF within

a layer and SNOW refers to a flux of ice between layers.

The structure is to loop downwards from the top of the model, calculating new values of vapour, liquid and ice at each level by the calculation of transfer terms. Falling water quantities, that is rain and a portion of the ice in each level, are considered as fluxes (in units of $\text{kg m}^{-2} \text{s}^{-1}$) at the bottom of the layer from which they were calculated.

The transfer terms between Q, QCL, QCF and RAIN are in the following order:

Fall of ice into and out of the current layer (QCF \rightarrow QCF);
 Homogenous nucleation of ice from liquid (QCL \rightarrow QCF);
 Heterogenous nucleation of ice from liquid or vapour (QCL \rightarrow QCF, Q \rightarrow QCF) ;
 Deposition of ice from liquid or vapour (QCL \rightarrow QCF, Q \rightarrow QCF);
 Hallett Mossop process (QCL \rightarrow QCF, Q \rightarrow QCF) *This term is switched off*;
 Evaporation of ice to vapour (QCF \rightarrow Q);
 Riming (QCL \rightarrow QCF);
 Capture of raindrops by ice particles (RAIN \rightarrow QCF);
 Evaporation of melting ice (QCF \rightarrow Q);
 Melting of ice to rain (QCF \rightarrow RAIN);
 Evaporation of rain (RAIN \rightarrow Q);
 Accretion of water droplets by rain (QCL \rightarrow RAIN);
 Autoconversion of liquid to rain (QCL \rightarrow RAIN).

Each of these terms will be considered separately below. Temperature increments in the layer due to latent heat release are applied for each term which results in a change of phase, although these are not detailed below. The temperature changes take the form of $\Delta T = L_s/c_p [\Delta Q \rightarrow \text{QCF}] + L_v/c_p [\Delta Q \rightarrow \text{QCL}] + L_f/c_p [\Delta Q \rightarrow \text{QCF}]$ where L_s, L_v, L_f represent the latent heats of sublimation, vaporization and melting (in J kg^{-1}) and c_p (in $\text{J K}^{-1} \text{kg}^{-1}$) is the heat capacity of air at constant pressure. The calculations are carried out in the **LSP_ICE** subroutine. The appendix provides a flow chart of the structure of the scheme.

PARAMETERIZATION OF RAIN AND ICE.

Real rain and ice particle distributions can be extremely varied. The model attempts to parameterize them using a number of parameters, which have fixed values in the 3A scheme.

The drop size distribution for rain is parameterized by:

$$N_{rain}(D_{rain}) = (X1R \Lambda^{X2R}) \exp(-\Lambda D_{rain}) \quad (\text{P26.63})$$

where $X1R$ and $X2R$ are parameters specified by the scheme, D_{rain} is the diameter of each raindrop (in m) and Λ depends on the rainfall rate. If $X2R = 0$ and $X1R = 8 \times 10^6 \text{ m}^{-4}$ then a Marshall Palmer (1948) raindrop distribution is modelled. The fall speeds of raindrops at an air density of 1 kg m^{-3} are parameterized by:

$$v_{rain}(D_{rain}) = CR D_{rain}^{DR} \quad (P26.64)$$

where CR and DR are parameters specified by the scheme. The particle size distribution for ice is given by a gamma distribution, similar equation to (P26.63) if $X4I=0$:

$$N_{ice}(D_{ice}, T) = f(T) (X1I \Lambda_{ice}^{X2I}) D_{ice}^{X4I} \exp(-\Lambda_{ice} D_{ice}) \quad (P26.65)$$

where $X1I$, $X2I$ and $X4I$ are parameters specified by the scheme, T is the temperature and $f(T)$ is a specified function of temperature. D_{ice} represents the equivolume diameter of the ice particle. The form of $f(T)$ is as follows:

$$f(T) = \exp\left(\frac{-X3I T/^\circ C}{8.18/^\circ C}\right) \quad (P26.66)$$

where $X3I$ is specified by the scheme. If $X3I=0$ then there is no temperature dependence of N_{ice} . $f(T)$ is written as TCG in the equations which follow. The fall velocity for an ice particle at an air density of 1 kg m^{-3} is given by a similar equation to that for rain:

$$v_{ice}(D_{ice}) = CI D_{ice}^{DI} \quad (P26.67)$$

where CI and DI are specified by the scheme. An additional equation is required for ice since the density of ice particles is not that of liquid water but can vary. The mass diameter relationship is given by:

$$m_{ice}(D_{ice}) = AI D_{ice}^{BI} \quad (P26.68)$$

where AI and BI are specified by the scheme.

The quantities AI , BI , CI , DI , $X1I$, $X2I$, $X3I$, $X4I$, CR , DR , $X1R$ and $X2R$ take the following values in version 3A:

$$\begin{aligned} X1R &= 8.0 \times 10^6 \text{ m}^{-4} \\ X2R &= 0 \\ X1I &= 2.0 \times 10^6 \text{ m}^{-4} \\ X2I &= 0 \\ X3I &= 1.0 \\ X4I &= 0 \\ AI &= 0.069 \text{ kg m}^{-2} \\ BI &= 2.0 \\ CI &= 25.2 \text{ m}^{0.473} \text{ s}^{-1} \\ DI &= 0.527 \\ CR &= 386.8 \text{ m}^{0.33} \text{ s}^{-1} \\ DR &= 0.67. \end{aligned}$$

The raindrop size distribution is from Marshall and Palmer (1948). The raindrop fall speeds are from Sachidananda and Zrnić (1986). The ice distribution is similar to that used by Cox (1988). The ice fall speeds and mass distribution are a tuned version of Cox (1988) and give a similar result to Heymsfield (1977).

The model transfer terms are not written in terms of individual particles but in terms of bulk quantities such as ice water content and rainfall flux. The method is to infer the distribution of ice (or rain) from the content (or flux) using the above parameters (except for autoconversion, which follows Tripoli and Cotton (1980)). The microphysical rate equations governing the transfer of mass to or from each particle are solved, and the rate across the distribution is integrated to yield a rate for the bulk quantity in a similar way to Rutledge and Hobbs (1983). Since the integrals over the distribution can be analytically solved, or where they cannot approximations can be applied, each transfer term can be written down as a function of not only the bulk quantities but the above parameters. This is described in the bulk of this paper if the reader requires specific details.

NOTE ON SUB GRID SCALE TREATMENT.

The sub grid scale treatment is an important part of the scheme. The transfer calculations are formulated assuming that there is no variability across the gridbox. This is because the real situation is immensely complex - overlaps between each water quantity must be considered as well as the distribution of each quantity across the grid box and changes in temperature. The terms are then altered in order to account for some degree of sub grid scale change across the grid box. The changes are considered for each term below, but act so that ice cannot form until a critical grid box mean relative humidity has been reached.

There are three parameters specifying the sub grid scale nature. RATEQ represents a multiplication factor to the rate terms (equal to 1.0 in the 3B scheme), RATEQS is the relative humidity at which deposition can start to occur (set to a function of the critical grid box mean relative humidity, RH_{crit} , and the cloud fraction) and RATEQCF represents a reduction in rate proportional to the amount of ice present (set to 0.0). RATEQS is defined in version 3B as:

$$RATEQS = RH_{crit} (1 - cf) + 1 \text{ cf}$$

where cf is the cloud fraction (liquid or ice fraction, depending upon the transfer term). This formulation will give RATEQS as 1 if the cloud fraction is one, and RH_{crit} if it is zero. (The 3A scheme uses values of RATEQ=0.5, RATEQS= RH_{crit} , RATEQCF=1.0).

A detailed description of the scheme now begins. The physics and mathematics gets complicated. If only an overview of the scheme is required, you are not recommended to read any further!

THE MICROPHYSICS SCHEME.

The microphysics scheme (**LSP_ICE**) is called for each model level (from **LSPPN3A**), looping from the top downwards. The points which are called are those where the total cloud fraction is greater than a small minimum threshold (0.001), or where there is

rain or snow falling from the layer above, or where the ice content is greater than 0. Hence there must be liquid cloud present, and hence a minimum relative humidity before ice cloud can form from other water phases, although this is not restrictive. Ice can, of course, also be advected into a region by the dynamics routine, but this is not discussed in this paper.

A loop is started over the condensed points which are being considered. Within the points loop the microphysical transfer terms are carried out in the order shown in the introduction. Each of the transfer terms acts sequentially, so using the values of QCL, QCF etc. *after* the previous transfer term has acted as inputs to the next transfer term. After the end of the loop there are some tidying up terms which act to remove residual ice. New values of vapour (Q), liquid (QCL), ice (QCF) and temperature (T) for that level form the output values. Rainfall (RAIN), snowfall (SNOW) and fall velocity of the snow (VF) are passed to the next level down. The appendix details the layout of the microphysics in terms of a flow chart.

Each section of the microphysics code is now dealt with in turn.

SETTING UP.

Several combinations of the ice particle and raindrop distribution constants AI , BI , etc. appear often throughout the calculation of the transfer terms. They are defined below and are calculated at the start of each call to the large scale precipitation scheme:

$$\text{CONSTP}(1) = 1 / (AI \ X1I \ \Gamma(\text{BI}+1+\text{X4I}))$$

where $\Gamma(\text{BI}+1+\text{X4I})$ represents the gamma function of $(\text{BI}+1+\text{X4I})$;

$$\text{CONSTP}(2) = 2 \ \pi \ X1R \ ;$$

$$\text{CONSTP}(3) = CI \ \Gamma(\text{BI}+\text{DI}+1+\text{X4I}) / \Gamma(\text{BI}+1+\text{X4I}) \ ;$$

$$\text{CONSTP}(4) = \pi \ X1I \ CI \ \Gamma(\text{DI}+3+\text{X4I}) / 4 \ ;$$

$$\text{CONSTP}(5) = 2 \ \pi \ X1I \ ;$$

$$\text{CONSTP}(6) = 0.44 \ Sc^{1/3} \ CI^{1/2} \ \Gamma((\text{DI}+5+2 \ \text{X4I})/2) / \mu^{1/2}$$

where Sc is the Schmidt number and has value 0.6 and μ is the dynamic viscosity of air at a temperature of 273K and has the value $1.717 \times 10^{-5} \text{ kg m}^{-1} \text{ s}^{-1}$;

$$\text{CONSTP}(7) = 2\pi \ k_a \ X1I / L_f$$

where k_a is the thermal conductivity of air at a temperature of 273K and has the value of $2.40 \times 10^{-2} \text{ J m}^{-1} \text{ s}^{-1} \text{ K}^{-1}$ and L_f is the latent heat of fusion of water and has the value of $0.334 \times 10^6 \text{ J kg}^{-1}$;

$$\text{CONSTP}(8) = \pi \ \rho_{\text{wat}} \ X1R \ \Gamma(\text{DR}+4) \ CR / 6$$

where ρ_{wat} is the density of liquid water and has the value of 1000 kg m^{-3} ;

$$\text{CONSTP}(9) = \pi^2 \ \rho_{\text{wat}} \ X1I \ X1R \ ;$$

$$\text{CONSTP}(10) = \pi \ X1R \ CR \ \Gamma(\text{DR}+3) / 4 \ ;$$

$$\text{CONSTP}(11) = CR \ \Gamma(\text{DR}+4) \ ;$$

$$\text{CONSTP}(12) = 0.31 \ Sc^{1/3} \ \Gamma((\text{DR}+5)/2) \ CR^{1/2} / \mu^{1/2} \ ;$$

$$\text{CONSTP}(13) = \Gamma(\text{X4I}+2) \ ;$$

$$\text{CONSTP}(14) = \Gamma(\text{X4I}+3) / 4 \ ;$$

$$\text{CONSTP}(15) = 2 \ \Gamma(\text{X4I}+2) \ ;$$

$$\text{CONSTP}(16) = 5 \ \Gamma(\text{X4I}+1) \ ;$$

$$\text{CX}(1) = (2 + \text{X4I} - \text{X2I}) / (\text{BI} + 1 + \text{X4I} - \text{X2I}) \ ;$$

$$\begin{aligned}
CX(2) &= (5 + DI + 2 X4I - 2 X2I) / (2 (BI + 1 + X4I - X2I)) ; \\
CX(3) &= DI / (BI + 1 + X4I - X2I) ; \\
CX(4) &= (3 + DI + X4I - X2I) / (BI + 1 + X4I - X2I) ; \\
CX(5) &= DR / (4 + DR - X2R) ; \\
CX(6) &= 1 / (X2I - X4I - 1 - BI) ; \\
CX(7) &= (3 + DR - X2R) / (4 + DR - X2R) ; \\
CX(8) &= X2I ; \\
CX(9) &= X2R ; \\
CX(10) &= 1 / (4 + DR - X2R) ; \\
CX(11) &= (DR + 5)/2 - X2R ; \\
CX(12) &= 2 - X2R ; \\
CX(13) &= X3I ; \\
CX(14) &= 3 + X4I ; \\
CX(15) &= 2 + X4I ; \\
CX(16) &= 1 + X4I .
\end{aligned}$$

These terms will appear frequently in the discussion that follows.

RAIN and SNOW are initialized to zero at each point for the highest level (so there is no precipitation falling into the top layer). The loop over points then begins. The density of air is calculated via the virtual temperature as:

$$\rho_{air} = \frac{p}{R T (1 + 0.6Q - Q_{CL} - Q_{CF})} . \quad (P26.69)$$

R is the gas constant for dry air and has the value of 287 J kg⁻¹ K⁻¹ and p is the pressure in N m⁻² (or Pa). The units of T are Kelvin. Then two correction factors are defined. The first represents the change in fall speeds of particles as a function of the density of air which can be written as (Pruppacher and Klett, 1980):

$$v_t = v_0 \left(\frac{\rho_0}{\rho} \right)^{0.4} , \quad (P26.70)$$

where v_t is the fall velocity at a given air density ρ and v₀ is the fall velocity at the reference air density of ρ₀ (1 kg m⁻³). The variable CORR represents the density variation above, so CORR = (ρ₀ / ρ)^{0.4}. There is also a variation of the thermal conductivity, viscosity and diffusivity of air on temperature. Theoretically these dependencies are identical and they are approximated by the expression:

$$k_a(T) = k_a(273) \left(\frac{T}{273} \right)^{\frac{3}{2}} \left(\frac{393}{T+120} \right) , \quad (P26.71)$$

where T is in Kelvin and k_a(273) is the thermal conductivity at 273K which has the value 2.40 x 10⁻² J m⁻¹ s⁻¹ K⁻¹. The variable CORR2 represents the temperature dependence of this equation, so CORR2 = (T / 273)^{3/2} (393 / (T + 120)). This also represents corrections to the viscosity and diffusivity of air due to temperature. Diffusivity also depends on pressure.

The saturated vapour pressures (both for liquid and ice) are calculated from the saturated mixing ratios using the dependence $e_{\text{sat}} = q_{\text{sat}} p / \epsilon$, where q_{sat} is the saturated mixing ratio (of liquid or ice) and $\epsilon = 0.622$.

After the setting up has been completed the actual transfer terms can be calculated and applied. These are presented in separate sections below, in the sequence they act in the code. For each transfer process the theoretical equation for one particle of diameter D is given first, then the bulk integral over the parameterized distribution is presented using XII , $X2I$ terms etc., and finally the simplified version using the CX and $CONSTP$ terms. In these derivations the expression linking the slope, Λ , of the ice particle size distribution to the ice water content, QCF , is frequently used. This is derived directly from the integral:

$$QCF = \frac{1}{\rho} \int_{D=0}^{\infty} N(D) m(D) dD, \quad (P26.72)$$

which, using the parameterizations of XII , $X2I$ etc. for ice, can be written as:

$$QCF = \frac{1}{\rho} \int_{D=0}^{\infty} XII TCG \Lambda^{X2I} D^{X4I} \exp(-\Lambda D) AI D^{BI} dD \quad (P26.73)$$

where TCG represents the temperature function $f(T)$ in (P26.66). This becomes on integration

$$QCF = \frac{XII TCG AI \Gamma(BI+1+X4I)}{\rho \Lambda^{-X2I+BI+1+X4I}}. \quad (P26.74)$$

Inverting this to give Λ in terms of QCF produces the expression:

$$\Lambda = \left(\frac{XII TCG AI \Gamma(BI+1+X4I)}{QCF \rho} \right)^{\frac{1}{-X2I+BI+1+X4I}}. \quad (P26.75)$$

This will be frequently used in the derivations below.

FALL OUT OF ICE TO THE NEXT LAYER.

Firstly the mass weighted fall velocity of the ice is calculated. This is theoretically defined as:

$$\bar{v}_t = \frac{\int_{D=0}^{\infty} v_t(D) m(D) N(D) dD}{\int_{D=0}^{\infty} m(D) N(D) dD}. \quad (P26.76)$$

Writing in terms of the parameters, this becomes:

$$\bar{v}_t = \frac{\int_{D=0}^{\infty} CORR CI D^{DI} AI D^{BI} X1I TCG \Lambda^{X2I} D^{X4I} \exp(-\Lambda D) dD}{\int_{D=0}^{\infty} AI D^{BI} X1I TCG \Lambda^{X2I} D^{X4I} \exp(-\Lambda D) dD} \quad (P26.77)$$

This can be integrated to give a function of Λ .

$$\bar{v}_t = CORR CI \Lambda^{-DI} \frac{\Gamma(DI+BI+1+X4I)}{\Gamma(BI+1+X4I)} \quad (P26.78)$$

Inserting the relationship between Λ and ice water content in (P26.75) gives:

$$\bar{v}_t = CORR CI \frac{\Gamma(DI+BI+1+X4I)}{\Gamma(BI+1+X4I)} \left[\frac{QCF \rho}{X1I TCG AI \Gamma(BI+1+X4I)} \right]^{\frac{DI}{BI+1+X4I-X2I}} \quad (P26.79)$$

By writing in terms of CONSTP and CX values this can be reduced to:

$$\bar{v}_t = CORR CONSTP(3) \left[\frac{QCF \rho}{TCG} \frac{CONSTP(1)}{\Gamma(BI+1+X4I)} \right]^{CX(3)} \quad (P26.80)$$

This is the equation used to calculate the bulk fall speeds.

The advection of ice downwards is calculated by assuming that the amount of ice content which originates in the layer and falls through the bottom of the layer is given by the quantity $\Delta QCF = v_t QCF \Delta t / \Delta z$, where Δt is the timestep and Δz is the depth of the layer (m), assuming that this is not greater than the amount of ice in the layer to begin with (otherwise $\Delta QCF = QCF$). The amount which falls out of the layer is stored as a flux. The new ice content is then equal to the original ice content minus what has fallen out plus what has fallen in from the layer above (provided this does not fall completely through the layer). The velocity of the falling ice is stored. If the velocity of ice falling into a layer is greater than that required for the ice to fall completely through the layer in a single timestep (i.e. $v_t > \Delta z / \Delta t$) then some of the falling ice does not stay in this layer but falls to the layer below to be added to the contribution from the layer itself. The ice which stays has ice content $QCF = SNOW / (\rho v_t)$, and the rest of the ice is allowed to fall through. The velocity v_t which is output from this layer is then the maximum of that from the previous layer and that calculated in this layer. SNOW initially represents the flux into the layer, but is overwritten by the value of the flux out of the layer after this and the new ice water content in the layer have been calculated.

To convert from the change in ice content (kg kg⁻¹) to a change in flux (kg m⁻² s⁻¹) one needs to use the expression:

$$\Delta QCF = \frac{\Delta SNOW \Delta t}{\rho \Delta z} \quad (P26.81)$$

where Δt is the timestep (s) and Δz is the depth of each layer (m).

Hence the updating of QCF and SNOW can be summarized as:

If v_t (into layer) $\leq \Delta z/\Delta t$

$$QCF(t+\Delta t) = QCF(t) + \text{fall in} - \text{fall out}$$

$$SNOW(\text{out of layer}) = \text{fall out} \rho \Delta z/\Delta t$$

If v_t (into layer) $> \Delta z/\Delta t$

$$QCF(t+\Delta t) = SNOW(\text{into layer}) / [v_t (\text{into layer}) \rho]$$

$$SNOW(\text{out of layer}) = (\text{fall through} + \text{fall out}) \rho \Delta z/\Delta t$$

The sub grid scale nature of the problem is handled by using ‘in cloud’ ice water contents to calculate the fall speeds in (P26.80). QCF is replaced by QCF/CF_{ice} , where CF_{ice} is the ice cloud fraction.

NUCLEATION.

In order for ice to grow by the deposition mechanism, there must be ice present to begin with. There are two methods for nucleation, homogenous nucleation, which represents spontaneous freezing of liquid water drops at a low enough temperature, and heterogenous nucleation, which represents the conversion of liquid drops or vapour to ice through ice nuclei.

The homogenous term acts to freeze all liquid (QCL) with a temperature less than -40°C . The heterogenous nucleation is more involved.

The heterogenous nucleation term will seed a grid box with a small amount of ice depending upon the temperature and humidity of the grid box. If *all* the following criteria are met:

- 1) Liquid cloud fraction $> CF_{min}$ (where $CF_{min}=0.001$) or $QCF>0$
- 2) $Q > RATEQS Q_{sat\ ice}$ (where $RATEQS=RH_{crit}$)
- 3) $Q > Q_{sat\ liquid} RATEQS g(T)$

where $g(T)$ is the minimum of 1 and $(188.92 + 2.81 T/^\circ\text{C} + 0.013336 (T/^\circ\text{C})^2 - 10)/100$

- 4) $T < T_{nuc}$ where T_{nuc} is another specified parameter (-10°C)

then ice is allowed to be seeded. This is done by assuming the number of activated ice nuclei is a function of the temperature and assigning a small mass of 10^{-12} kg to each seed crystal (which is converted to an increment in mixing ratio). The number of nuclei produced per timestep is the minimum of $0.01 \exp(-0.06 T/^\circ\text{C})$ and 10^5 (Fletcher 1962).

This provides an increment of ice which is added to the ice water content at the expense of liquid water or, if no liquid is available, vapour, as long as $Q+QCL$ does not fall below the limits given by 2) and 3). The preferential transfer from liquid to ice rather than vapour to ice represents the Bergeron-Findeison process. There is no record kept of ice nuclei number so this term is considered at each timestep.

The criteria are chosen: to match the empirical conditions for cirrus cloud formation (condition 3); not to nucleate ice unless the temperature falls below a minimum value since clouds rarely contain ice unless their cloud top temperature is below -10°C (condition 4); and that ice cannot be formed if Q is not greater than $RH_{crit} Q_{sat\ ice}$ (sub grid variability - condition 2). The criterion 1 is required for the microphysics subroutine to be called in the first place

but is not restrictive.

At version 3A critereon 3) is replaced by: $Q > Q_{\text{sat liquid}} g(T)$.

DEPOSITION AND EVAPORATION OF ICE.

A single particle of diameter D will grow by vapour deposition at a rate given by the equation:

$$\dot{m} = \frac{4\pi D (S_i - 1) c f}{\left(\frac{L_s}{RT} - 1\right) \frac{L_s}{k_a T} + \frac{RT}{X e_{\text{sat ice}}}} \quad (\text{P26.82})$$

X is the diffusion coefficient of air, which is proportional to μ/p where μ is the dynamic viscosity of air, c is a capacitance term related to the fact that particles are not necessarily spherical, and f is a ventilation coefficient. $(S_i - 1)$ is the supersaturation factor with respect to ice. The dot over the 'm' represents a time derivative, d/dt . Although the particles could be a variety of shapes, the capacitance is assumed to be that of spherical particles, so $c=0.5$. To proceed, multiply both top and bottom of the growth equation by $e_{\text{sat ice}} T^2$. Note that X depends on temperature and pressure as: $X(T,p)=X(273 \text{ K}, 100 \text{ kPa}) \text{ CORR2} (100 \text{ kPa} / p)$. Also note that k_a depends on temperature as: $k_a = k_a(273 \text{ K}) \text{ CORR2}$.

Write the following groups of physical constants as:

$$\begin{aligned} \text{APB1} &= L_s^2 / (R k_a (273 \text{ K})) &&= 7.14 \times 10^{11} \text{ m s K}^2 \text{ kg}^{-1} \\ \text{APB2} &= L_s / k_a (273 \text{ K}) &&= 1.16 \times 10^8 \text{ m s K kg}^{-1} \\ \text{APB3} &= R / (100 \text{ kPa } X(273 \text{ K}, 100 \text{ kPa})) &&= 2.416 \times 10^2 \text{ m s K}^{-1} \text{ kg}^{-1}. \end{aligned}$$

Then obtain the equation in a simpler form of:

$$\dot{m} = \frac{4 D (S_i - 1) f \text{ CORR2 } T^2 e_{\text{sat ice}}}{\text{APB1 } e_{\text{sat ice}} - \text{APB2 } e_{\text{sat ice}} T + \text{APB3 } T^3 p} \quad (\text{P26.83})$$

Now write the denominator as APLUSB to further simplify the algebra. The ventilation coefficient is written as an approximation (for example, Pruppacher and Klett, 1978):

$$f(v) = 0.65 + 0.44 \text{ Sc}^{\frac{1}{3}} \text{Re}^{\frac{1}{2}}, \quad (\text{P26.84})$$

where the Schmidt number, Sc , takes the constant value of 0.6 and the Reynolds number is given by $\text{Re} = \rho D v(D) / \mu$. This gives the expression:

$$\dot{m} = \frac{4 D (S_i - 1) T^2 e_{\text{sat ice}} \text{CORR2}}{\text{APLUSB}} \left[0.65 + 0.44 \text{ Sc}^{\frac{1}{3}} \left(\frac{v \rho D}{\mu} \right)^{\frac{1}{2}} \right]. \quad (\text{P26.85})$$

The viscosity μ varies with temperature so the correction factor CORR2 should be used in the ventilation term. v varies with density, so use the CORR correction as well. Now write as an integral over the drop size distribution using the mass-diameter relationship and fall speed-diameter relationship given by the parameters AI , etc. Hence the rate of change of ice water

content is given by:

$$Q\dot{C}_F = \frac{1}{\rho} \frac{4 (S_i - 1) T^2 e_{satice} CORR2}{APLUSB} \quad (P26.86)$$

$$\int_{D=0}^{\infty} D [0.65 + 0.44 S_C \frac{1}{3} \frac{(CI D^{DI})^{\frac{1}{2}} CORR^{\frac{1}{2}} \rho^{\frac{1}{2}} D^{\frac{1}{2}}}{\mu (273)^{\frac{1}{2}} CORR2^{\frac{1}{2}}}] TCG X1 I \Lambda^{X2I} D^{X4I} \exp(-\Lambda D) dD.$$

This is then integrated to give the equation:

$$Q\dot{C}_F = \frac{4 (S_i - 1) T^2 e_{satice} CORR2 TCG X1 I \Lambda^{X2I}}{\rho APLUSB} \quad (P26.87)$$

$$\left[\frac{0.65 \Gamma (2+X4I)}{\Lambda^{2+X4I}} + \frac{0.44 S_C \frac{1}{3} CI^{\frac{1}{2}} CORR^{\frac{1}{2}} \rho^{\frac{1}{2}} \Gamma (\frac{5+DI}{2} + X4I)}{[\mu (273)]^{\frac{1}{2}} CORR2^{\frac{1}{2}} \Lambda^{\frac{5+DI}{2} + X4I}} \right].$$

Now write Λ in terms of QCF. To simplify further, use the variable:

$$PR02 = \left[\frac{\rho QCF CONSTP(1)}{TCG} \right]. \quad (P26.88)$$

This means that $\Lambda = PR02^{-1/(BI+1-X2I+X4I)}$ using (P26.75). Now write other factors in terms of CX and CONSTP to give the final coded expression.

$$Q\dot{C}_F = \frac{(S_i - 1) T^2 e_{satice} CORR2 TCG CONSTP(5)}{\rho APLUSB} \quad (P26.89)$$

$$\left[0.65 CONSTP(13) PR02^{CX(1)} + \frac{CONSTP(6) CORR^{\frac{1}{2}} \rho^{\frac{1}{2}} PR02^{CX(2)}}{CORR2^{\frac{1}{2}}} \right].$$

Again, growth in ice is at the expense of first the liquid (QCL) and then vapour (Q), assuming there is enough available moisture to grow the ice. Evaporation of ice is from the same calculation. The ice is sublimed directly to vapour.

Subgrid distribution:

The sub grid scale distribution of quantities needs to be considered since grid boxes are considerable in size. It is very difficult to obtain a description of how ice water contents, liquid contents, vapour contents and temperature all vary simultaneously across a grid box, and probably impossible to solve it analytically and quickly. Hence a much simpler approach

is used. Instead of the rate being proportional to the mean supersaturation across the gridbox, the supersaturation is replaced by the term:

$$(S_i - 1) = \left[\frac{Q + Q_{CL} - Q_{satice}}{Q_{satice}} \right] \rightarrow RATEQ \frac{(Q + Q_{CL} - RATEQS Q_{satice} - RATEQCF QCF)}{Q_{satice}} \quad (P26.90)$$

The parameters in the 3B scheme are: RATEQ=1.0; RATEQS=RH_{crit} (1-cf_{ice}) + 1 cf_{ice}; RATEQCF=0.0; where cf_{ice} is the ice cloud fraction. (The parameters in the 3A scheme are: RATEQ=0.5; RATEQS=RH_{crit}; RATEQCF=1.0.) This allows growth of ice when the gridbox mean relative humidity equals a critical value given by RH_{crit}, the same critical value as for the formation of liquid cloud. There is a limit imposed on the Q+QCL term so as not to exceed Q_{sat liq} (since in reality the supersaturation cannot exceed that for liquid as liquid would then condense out very rapidly).

Available Moisture:

The moisture available to such a subgrid parameterization is found by setting (S_i-1) = 0. Rearranging (P26.90) in this case and writing in terms of Q_t = Q+QCL+QCF, with the limit on Q+QCL removed, gives an ‘equilibrium’ value of QCF of:

$$QCF_{eq} = \frac{Q_t - RATEQS Q_{satice}}{RATEQCF + 1} \quad (P26.91)$$

So the available moisture for deposition is simply QCF_{eq}-QCF:

$$Moisture_{available} = \frac{Q + Q_{CL} - RATEQS Q_{satice} - RATEQCF QCF}{RATEQCF + 1} \quad (P26.92)$$

Parameterization of deposition.

If deposition occurs then the implementation of the parameterization is in a simple explicit form:

$$\Delta QCF = \min [QCF \Delta t, \text{available moisture}] \quad (P26.93)$$

If evaporation occurs then there is an additional limit of the ice water content (so no more QCF can be removed than was there to begin with).

The formulation of the deposition term is used in a similar way for the evaporation of melting snow, the melting of snow, and the evaporation of rain. Hence these terms are considered below in slightly less detail.

HALLETT MOSSOP PROCESS.

The extra deposition rate onto secondary ice particles generated by splintering is very crudely parameterized. It assumes that the rate of deposition can be increased by a factor proportional to the liquid water content of the cloud. Hence:

where QCL₀ is a scaling liquid water content (equal to 0.1 g kg⁻¹), f(T) is a function of

$$HM = (1 + f(T) \frac{QCL}{QCL_0})$$

temperature, and HM is a factor which multiplies the deposition rate, but not the sublimation rate. In version 3B, $f(T)$ is set to zero to switch off the process, but can be switched on by specifying three temperature parameters: the maximum and minimum temperatures for the process, and a decay rate in terms of K^{-1} . The function is calculated assuming that secondary ice particles which are produced between the specified temperature range are advected upwards and decay exponentially in number as the temperature decreases. Recommended values to use are $T_{min}=-8^{\circ}C$; $T_{max}=-3^{\circ}C$; and $T_{decay}=7$ K. The functionality for this process is not included at version 3A.

RIMING

This term represents the sweep out and subsequent freezing of liquid water droplets by falling ice particles. The term only acts if the temperature is below freezing. The liquid droplets are assumed to be stationary in the process and the ice particles are assumed to be represented by spheres of diameter D . The collection efficiency is assumed to be equal to 1. The expression for the collection of rime by one falling ice particle can be written as:

$$\dot{m} = \frac{\pi}{4} D^2 v(D) \rho Q_{CL}, \quad (\text{P26.94})$$

which represents the sweepout of all liquid water particles contained in a cylinder of area equal to the cross sectional area of the falling particle. This expression can be integrated in terms of the parameters XII etc. to give the following expression in terms of the rate of change of ice mixing ratio due to riming:

$$Q\dot{C}_F = \int_{D=0}^{\infty} \frac{\pi}{4} D^2 Q_{CL} CORR_{CI} D^{DI} XII \Lambda^{X2I} D^{X4I} TCG \exp(-\Lambda D) dD. \quad (\text{P26.95})$$

This can be integrated to give:

$$Q\dot{C}_F = \frac{\pi}{4} Q_{CL} CORR_{CI} XII \Lambda^{X2I} TCG \frac{\Gamma(3 + DI + X4I)}{\Lambda^{3+DI+X4I}}. \quad (\text{P26.96})$$

This is the equation used in the solution of the term. The term is solved implicitly in terms of the liquid water content, since the rate is proportional to the liquid water content. Hence the new liquid water content is solved as $Q_{CLNEW} = Q_{CL} / (1 + \alpha \Delta t)$ where $dQ_{CF}/dt = \alpha Q_{CL}$. There is no attempt made to consider sub grid scale variations in this term. This is because of the complexity of specifying the overlaps between liquid and ice. It is assumed in this term that both liquid and ice are evenly spread around the gridbox.

Hence the final expression for the new liquid water content is, replacing terms by $CONSTP$ and CX ,

$$Q_{CL_{new}} = \frac{Q_{CL}}{1 + CONSTP(4) TCG CORR_{\Delta} t \left[\frac{\rho Q_{CF} CONSTP(1)}{TCG} \right]^{CX(4)}}. \quad (\text{P26.97})$$

Q_{CF} and T are then altered in order to conserve moisture and energy.

CAPTURE.

This term represents the freezing of supercooled rain by collision with snow. This is a process which rarely happens in large scale systems and its effect is generally small, so it could be left out of the code if necessary. However, the coding involves the following method.

For collisions between raindrops of diameter D_R and ice particles of diameter D_i the sweepout rate is given by the integral:

$$Q\dot{C}^F = \frac{1}{\rho} \int_{D_R=0}^{\infty} \int_{D_i=0}^{\infty} N_i(D_i) N_R(D_R) E(D_i, D_R) \frac{\pi}{4} (D_i + D_R)^2 \times \quad (P26.98)$$

$$|v(D_i) - v(D_R)| \frac{\pi}{6} \rho_{\text{wat}} D_R^3 dD_i dD_R.$$

$N_i(D_i)$ is the particle size distribution for ice and $N_R(D_R)$ is the drop size distribution for rain. This is a sweepout rate like for riming (P26.95) except that both sets of colliding particles can fall and have non zero diameters. Hence the sweepout area is $\pi(D_i + D_R)^2/4$ and the sweepout velocity is $|v(D_i) - v(D_R)|$ for each pair of particles. Now make the assumption that the collision and collection efficiency, $E(D_i, D_R)$, equals 1. The modulus of the velocity makes this equation extremely difficult to integrate precisely, so take out to the front of the equation a function of v_i and v_R ($f(v_i, v_R)$) which will approximate the velocity dependence. Hence the integral becomes:

$$Q\dot{C}^F = \frac{\pi^2}{24} \frac{X1I\Lambda_i^{X2I} TCG X1R\Lambda_R^{X2R} \rho_{\text{wat}} f(v_i, v_R)}{\rho} \times \quad (P26.99)$$

$$\int_{D_R=0}^{\infty} \int_{D_i=0}^{\infty} D_i^{X4I} (D_i + D_R)^2 D_R^3 \exp(-\Lambda_R D_R) \exp(-\Lambda_i D_i) dD_i dD_R.$$

Λ_R is the slope of the rain distribution and Λ_i is the slope of the ice distribution. This integrates to the expression:

$$Q\dot{C}^F = \frac{\pi^2 \rho_{\text{wat}} f(v_R, v_i) X1I\Lambda_i^{X2I} TCG X1R\Lambda_R^{X2R}}{24\rho} \times \quad (P26.100)$$

$$\left[\frac{\Gamma(3+X4I)\Gamma(4)}{\Lambda_i^{3+X4I}\Lambda_R^4} + \frac{2\Gamma(2+X4I)\Gamma(5)}{\Lambda_i^{2+X4I}\Lambda_R^5} + \frac{\Gamma(1+X4I)\Gamma(6)}{\Lambda_i^{1+X4I}\Lambda_R^6} \right].$$

To simplify the algebra, write the quantity in square brackets as LAMFAC1. Now consider the function of the mean velocities of rain and ice, v_R and v_i . When one of v_R or v_i is considerably greater than the other, one can approximate the integral of their modulus difference as the difference between their means. At smaller differences, another formulation must be chosen. If the difference in the mean velocities is less than a quarter of the average velocity, then the velocity difference considered will be a quarter of the average velocity. This is an extremely approximate, and in some degree arbitrary, solution to the problem. The function f is given the variable name FV1.

$$\begin{aligned} f(v_i, v_R) &= |v_i - v_R| && \text{if } |v_i - v_R| > (v_i + v_R)/8 \\ f(v_i, v_R) &= (v_i + v_R)/8 && \text{if } |v_i - v_R| \leq (v_i + v_R)/8 \end{aligned} \quad (P26.101)$$

To proceed, expressions must be obtained for v_R , v_i , Λ_R and Λ_i . The ice terms have

already been considered in (P26.80) and (P26.75). The corresponding terms for rain are found by a similar method:

$$RAIN = \int_{D=0}^{\infty} \frac{\pi}{6} D^3 \rho_{\text{wat}} N(D) v(D) dD. \quad (\text{P26.102})$$

This, when integrated and rearranged, gives the slope of the distribution in terms of the rainfall flux:

$$\Lambda_R = \left(\frac{CONSTP(8) CORR}{RAIN} \right)^{CX(10)}. \quad (\text{P26.103})$$

The mass weighted fall velocity can be written as:

$$V_R = \frac{\int_{D=0}^{\infty} \frac{\pi}{6} D^3 \rho_{\text{wat}} N(D) v(D) dD}{\int_{D=0}^{\infty} \frac{\pi}{6} D^3 \rho_{\text{wat}} N(D) dD}, \quad (\text{P26.104})$$

which gives, using the expression for Λ_R above,

$$\hat{v}_R = \frac{CORR}{6} CONSTP(11) \left[\frac{RAIN}{CONSTP(8) CORR} \right]^{CX(5)}. \quad (\text{P26.105})$$

Write LAMR1 and LAMS1 to represent the calculated values of Λ_R and Λ_i . Hence the solution for the rate of change of ice content is given by:

$$Q\dot{C}F = \frac{CONSTP(9)}{\rho} FV1 LAMR1^{CX(9)} TCG LAMS1^{CX(8)} LAMFAC1. \quad (\text{P26.106})$$

The transfer of moisture is then calculated explicitly, limited by the amount of RAIN available to be frozen. So the transfer in a timestep is $dQCF/dt \Delta t$. To convert RAIN into an equivalent water content uses the knowledge that all the rain falls from one level to the next in a timestep (it is diagnostic). So to conserve moisture, the equivalent water content for RAIN is expressed as $RAIN_{\text{mass}} = RAIN \Delta t / (\rho \Delta z)$ where Δz is the layer thickness like in (P26.81).

EVAPORATION OF MELTING SNOW.

This follows the formulation for deposition, except that the saturated vapour pressures for liquid water are used instead of those for ice. Hence we need to solve, from (P26.82):

$$\dot{m} = \frac{4 \pi D (S_w - 1) c f}{\left(\frac{L_v}{RT} - 1 \right) \frac{L_v}{k_a T} + \frac{RT}{X e_{\text{satwat}}}}, \quad (\text{P26.107})$$

where L_v is the latent heat of vaporization of water and equals $2.5 \times 10^6 \text{ J kg}^{-1}$ and $S_w - 1$ is

the supersaturation with respect to liquid water. This solves like (P26.89) as:

$$Q\dot{C}F = \frac{(S_w - 1) T' e_{satwat} CORR2 TCG CONSTP(5)}{\rho PR04} \quad x \quad (P26.108)$$

$$[0.65 CONSTP(13) PR02^{CX(1)} + \frac{CONSTP(6) CORR^{\frac{1}{2}} \rho^{\frac{1}{2}} PR02^{CX(2)}}{CORR2^{\frac{1}{2}}}] ,$$

where

$$PR04 = APB4 e_{satwat} - APB5 T e_{satwat} + APB6 T^3 p, \quad (P26.109)$$

like the bottom line of (P26.83). The values of the APB constants are:

$$\begin{aligned} APB4 &= L_v^2 / (R k_a(273)) = 5.57 \times 10^{11} \text{ m s K}^2 \text{ kg}^{-1} \\ APB5 &= L_v / k_a(273) = 1.03 \times 10^8 \text{ m s K kg}^{-1} \\ APB6 &= R / (X 100 \text{ kPa}) = 2.04 \times 10^2 \text{ m s K}^{-1} \text{ kg}^{-1} \end{aligned}$$

The sub grid scale calculations are performed like in the deposition term, so that (S_w-1) is calculated using (P26.90) except that $Q_{sat liq}$ replaces $Q_{sat ice}$.

The transfer is solved implicitly and is slightly involved. Define α by the ratio of the rate of change of moisture at the start of the timestep to the initial moisture available to be transferred. Then implicitly solve for the transfer as a function of α . This gives the solution:

$$\alpha = \frac{Q\dot{C}F \Delta t RATEQ}{(S_w-1) Q_{sat liq}} \quad (P26.110)$$

$$Transfer = \frac{\alpha}{1 + \alpha (1 + RATEQCF)} [Q + QCL - RATEQS Q_{sat liq} - RATEQCF QCF] . \quad (P26.111)$$

If α is small then this reduces to $Transfer = dQCF/dt \Delta t RATEQ$. If α is large then this reduces to the available moisture (similar to that defined in the deposition term P26.92).

MELTING OF SNOW.

This is very similar in structure to the deposition term. The equation being solved in this case is, for a single particle:

$$\dot{m} = -4\pi D C f \left[\frac{k_a}{L_m} (T_w - T_0) \right], \quad (P26.112)$$

where f is a ventilation factor, T_w is the wet bulb temperature and the term only applies if the wet bulb temperature is greater than 0°C . L_f is the latent heat of melting = $0.334 \times 10^6 \text{ J kg}^{-1}$. The minus sign signifies a reduction in the amount of ice. During melting, spherical particles

are assumed so the capacitance equals 0.5. Hence this has exactly the same form as the deposition term (P26.89), with the exception that the multiplying factors are different.

$$\frac{4(S_i - 1) T^2 e_{satice}}{APLUSB} \rightarrow \frac{2\pi k_a (T_w - T_0)}{L_m} \quad (P26.113)$$

Hence the rate can be written quite simply, like (P26.89), as:

$$Q\dot{C}F = -(T_w - T_0) \frac{TCG \text{ CONSTP}(7) \text{ CORR2}}{\rho} \times \quad (P26.114)$$

$$\left[0.65 \text{ CONSTP}(13) \text{ PRO2}^{CX(1)} + \frac{\text{CONSTP}(6) \text{ CORR}^{\frac{1}{2}} \text{ PRO2}^{CX(2)}}{\text{CORR2}^{\frac{1}{2}}} \right]$$

where CORR2 contains the dependence of k_a on T.

The wet bulb temperature needs to be defined. This is calculated using a numerical approximation which depends on the pressure and the temperature. This approximation is:

$$T_w = T - (Q_{satwat} - Q - QCL) (1329.31K + \quad (P26.115)$$

$$0.0074615 \text{ Km}^2 \text{ N}^{-1} (p - 0.85 \times 10^5 \text{ Nm}^{-2}) - 40.637 (T - 275K))$$

This is simply adjusting the dry bulb temperature with a linear function of the saturation deficit adjusted by the temperature and pressure.

The sub grid scale approximations used here are to the saturation deficit. At 3B, the term $(S_w - 1)$ is replaced by $(\text{RATEQS } Q_{satwat} - Q - QCL) / Q_{satwat}$ as in the deposition term (P26.90), where $\text{RATEQS} = \text{RH}_{crit} (1 - cf_i) + 1 cf_i$. (For 3A, $(S_w - 1)$ is replaced by $\text{RATEQ} (\text{RATEQS } Q_{satwat} + \text{RATEQCF } QCF - Q - QCL) / Q_{satwat}$ where $\text{RATEQ} = 0.5$, $\text{RATEQS} = \text{RH}_{crit}$ and $\text{RATEQCF} = 1$).

The rate is proportional to the temperature difference between the wet bulb temperature and the melting temperature so this equation is solved implicitly in terms of temperature. This gives:

$$\text{Temperaturechange} = (T_w - T_0) \left[1 - \frac{1}{1 + Q\dot{C}F \Delta t \frac{L_f}{C_p}} \right] \quad (P26.116)$$

The change in temperature is then related to the amount of ice melted by the latent heat of melting and the heat capacity of air.

EVAPORATION OF RAIN.

Again, this is very similar to deposition. The only differences are that $Q_{\text{sat wat}}$ is used instead of $Q_{\text{sat ice}}$, latent heats of vaporization are used instead of sublimation, spherical particles are considered, a different approximation to the ventilation coefficient is used (although of the same form) and the expression is in terms of a flux and not a mass.

For a single raindrop, the evaporation equation is given by an equation similar to (P26.82):

$$\dot{m} = \frac{4\pi D c f(S_w - 1)}{\left[\left(\frac{L_v}{RT} - 1 \right) \frac{L_v}{k_a T} + \frac{RT}{X e_{\text{sat wat}}} \right]} \quad (\text{P26.117})$$

Because the particles are spherical, the capacitance term equals 0.5. By multiplying top and bottom by $T^2 e_{\text{sat wat}}$, writing the new bottom line as PR04, and integrating over the distribution defined by XIR etc. one can write the rate of change of mass contained in the rain as:

$$RAIN_{mass} = \frac{2\pi XIR \Lambda^{XIR} (S_w - 1) T^2 e_{\text{sat wat}}}{PR04} \times \quad (\text{P26.118})$$

$$\int_{D=0}^{\infty} D \left[0.78 + 0.31 Sc^{\frac{1}{3}} \frac{(CR D^{DR})^{\frac{1}{2}} CORR^{\frac{1}{2}} \rho^{\frac{1}{2}} D^{\frac{1}{2}}}{\mu^{\frac{1}{2}} (273) CORR^{\frac{1}{2}}} \right] dD.$$

RAIN_{mass} is the mass of rain water per metre cubed [kg m⁻³]. This can be integrated in a similar way to before. Replacing terms with CONSTP and CX gives the result:

$$RAIN_{mass} = \frac{(S_w - 1) T^2 e_{\text{sat wat}} CONSTP(2) CORR2}{PR04} \times \quad (\text{P26.119})$$

$$\left(\frac{0.78}{LAMR1^{CX(12)}} + \frac{CONSTP(12) \rho^{\frac{1}{2}} CORR^{\frac{1}{2}}}{LAMR1^{CX(11)} CORR2^{\frac{1}{2}}} \right),$$

where the value LAMR1 represents the value of Λ_R and is given by (P26.103) in the capture section. Compare this result to (P26.89). The term is calculated implicitly in terms of subsaturation. Hence the transfer is proportional to $\alpha(S_w - 1)/(1 + \alpha)$ where α is a rate. The sub grid scale calculation is done in a similar way to that in the deposition term. Instead of calculating the subsaturation factor in terms of grid box averages there is a modification applied.

At 3B, RATEQ=1.0 and RATEQS=RH_{crit} cf_w + 1 cf_w where cf_w is the liquid cloud fraction.

$$-(S_w - 1) = \frac{(Q_{satwat} - Q - QCL)}{Q_{satwat}} \rightarrow \frac{RATEQ (RATEQS Q_{satwat} - Q - QCL)}{Q_{satwat}}. \quad (P26.120)$$

(RATEQ=0.5 and RATEQS=RH_{crit} in the 3A version.) This means that evaporation cannot proceed if it would cause cloud to form in the layer (in the absence of any ascent or cooling).

Since rain is treated as a diagnostic, the relationship between the change of rain mass and the inferred flux out of the layer (in order to conserve water) must be given by the expression (like P26.81):

$$\Delta RAINmass = \frac{\Delta RAIN \Delta t}{\rho \Delta z}. \quad (P26.121)$$

Δt is the model timestep and Δz is the thickness of the layer. RAIN is the flux of rain falling from this layer in kg m⁻² s⁻¹. Again, the transfer is implicit. Writing

$$\alpha = \frac{RAINmass RATEQ}{(S_w - 1) Q_{satliq}} \quad (P26.122)$$

gives the transfer as (using max(RATEQS Q_{sat liq} - Q - QCL , 0) as the available moisture):

$$Transfer = \frac{\alpha \Delta t}{1 + \alpha \Delta t} (availablemoisture). \quad (P26.123)$$

ACCRETION OF CLOUD BY RAIN.

This term is like the riming term in most respects. The volume swept out by one raindrop is:

$$\dot{m} = \frac{\pi}{4} D^2 v(D) \rho QCL. \quad (P26.124)$$

This is identical to (P26.94) for ice. This is written as an integral:

$$RAINmass = \int_{D=0}^{\infty} \frac{\pi}{4} D^2 QCL CORR CRD^{DRX1R} \Lambda^{X2R} \exp(-\Lambda D) dD. \quad (P26.125)$$

This is like (P26.95) for ice. Integrating and replacing Λ in terms of the rain rate gives the result:

$$RAINmass = QCL CONSTP(10) CORR PRO2^{CX(7)}, \quad (P26.126)$$

where

This is solved implicitly in terms of QCL, and the change in the mass of rain converted to

$$PRO2 = \frac{RAIN}{CONSTP(8) CORR}. \quad (P26.127)$$

a rain flux like in the evaporation term.

$$QCLNEW = \frac{QCL}{1 + \frac{RAINmass\Delta t}{QCL}}. \quad (P26.128)$$

AUTOCONVERSION.

This is a very ill-defined quantity. It is the amount of cloud water which converts itself to rain in a timestep due to collision-coalescence growth mechanisms. It is the initial source of precipitation in clouds whose tops never get to a temperature below 0°C, and hence acts to limit the liquid water contents of clouds. It can act at all temperatures. The theoretical equation is based not on the parameters that the rest of the transfer terms use, but on the equation of Tripoli and Cotton (1980). They write the rate of autoconversion as:

$$Q\dot{C}L = f_c QCL, \quad (P26.129)$$

provided QCL is greater than a limiting quantity derived below. The quantity f_c is supposed to represent the collision frequency between droplets. They write this as:

$$f_c = \pi r_c^2 E_c V_c N_c, \quad (P26.130)$$

where r_c is the radius of a typical water droplet, E_c is the collision efficiency, N_c is the number of droplets per metre cubed (assumed to be fixed at $3 \times 10^8 \text{ m}^{-3}$), and V_c is the Stokes terminal velocity of a particle. The Stokes velocity is given by:

$$V_c = \frac{4\rho_{wat} r_c^2 g}{18 \mu}. \quad (P26.131)$$

g is the acceleration due to gravity (9.81 m s^{-2}). Collecting these equations together and writing the radius, r_c , in terms of QCL, N_c , ρ and ρ_{wat} gives the expression to be used in the code:

$$Q\dot{C}L = \frac{4\pi}{18 \left[\frac{4}{3}\pi \right]^{\frac{4}{3}}} \frac{g E_c \rho^{\frac{4}{3}} QCL^{\frac{7}{3}}}{\mu (N_c \rho_{wat})^{\frac{1}{3}}}. \quad (P26.132)$$

The limit is considered to be when the drop radius is a critical size.

$$QCL_{limit} = \frac{4}{3}\pi \frac{\rho_{wat} r_{crit}^3 N_c}{\rho}. \quad (P26.133)$$

Autoconversion cannot proceed past this limit. To deal with the subgrid nature of this term, the in cloud values of liquid water content are used, so QCL is replaced by QCL / CF_{liquid} where CF_{liquid} is the liquid cloud fraction. Autoconversion is extremely non linear and this may not be a good enough way of dealing with the subgrid nature, but the other approximations which go into the equations are probably greater restrictions. Version 3B assumes that $g=9.81 \text{ m s}^{-2}$, $E_c=0.55$, $\mu(273 \text{ K})=1.717 \times 10^{-5} \text{ kg m}^{-1} \text{ s}^{-1}$, $\rho_{\text{wat}}=1000 \text{ kg m}^{-3}$, and $r_c=7 \times 10^{-6} \text{ m}$. These values are recommended by Tripoli and Cotton. The number concentration is assumed to be $6.0 \times 10^8 \text{ m}^{-3}$ over land and $1.5 \times 10^8 \text{ m}^{-3}$ over sea. This is consistent with the values specified in the radiation code. Extra parameters multiply the rate and limit equations to allow for simple tuning (although these extra parameters are set to 1 in the 3B scheme).

Version 3A writes N_c as $3 \times 10^8 \text{ m}^{-3}$, with $g=9.81 \text{ m s}^{-2}$, $E_c=0.55$, $\mu(273 \text{ K})=1.717 \times 10^{-5} \text{ kg m}^{-1} \text{ s}^{-1}$ and $\rho_{\text{wat}}=1000 \text{ kg m}^{-3}$, and we assume $r_c=7 \times 10^{-6} \text{ m}$ then the rate of change of QCL becomes:

$$Q\dot{C}L = 4.85 [m^4 kg^{-\frac{4}{3}} s^{-1}] \frac{\rho^{\frac{4}{3}} QCL^{\frac{7}{3}}}{CORR2}, \quad (\text{P26.134})$$

This is implemented explicitly:

$$Transfer = Q\dot{C}L \Delta t. \quad (\text{P26.135})$$

The limit becomes:

$$QCL_{\text{limit}} = \frac{4.31 \times 10^{-4} [kg m^{-3}]}{\rho}. \quad (\text{P26.136})$$

No account is made of any change in N_c or r_c with air mass in version 3A. This term remains highly tunable at best. The 'tuned' values used in version 3A are $4.87 \text{ m}^4 \text{ kg}^{-4/3} \text{ s}^{-1}$ for the rate constant (instead of $4.85 \text{ m}^4 \text{ kg}^{-4/3} \text{ s}^{-1}$ above) and $1.67 \times 10^{-4} \text{ kg m}^{-3}$ (instead of $4.31 \times 10^{-4} \text{ kg m}^{-3}$ above).

FINISHING THE ROUTINE.

The autoconversion term is the last term which is applied in the microphysics. A couple of small tidying up changes are then made.

The snow which falls out of one layer will not see the melting term until the next layer down. This can have the effect, especially if a long timestep is used, that falling snow is passed from layer to layer without seeing any of the microphysics terms. This can result in ice which falls from tropical cirrus cloud reaching the ground without melting! Hence at the end of the microphysics the SNOW sees an extra melting term. It does not use a rate but at 3A will melt as much snow as possible while keeping the wet bulb temperature (P26.115) above 0°C . At 3B a proportion of the SNOW will be melted, equal to the minimum of $v_f \Delta t / \Delta z - 1$ and 1, where v_f is the fall speed of the SNOW, Δt is the timestep, and Δz is the layer

depth. The equation P26.81 can convert the snow into an equivalent mixing ratio and the energy to melt the snow (and so the temperature increment) comes from the current layer. Any SNOW left over will be allowed to fall into the next layer down, to allow the lowering of the melting layer.

The final stage is to remove any tiny ice amounts which are left. If ice does not make up a significant fraction of the water content (determined at 3A by whether the ice content is less than the heterogenous nucleation increment (or m_0) and less than $1 \times 10^{-5} Q_{\text{sat ice}}$) it is sublimed to vapour. At 3B the threshold ice value is $1 \times 10^{-8} \text{ kg kg}^{-1}$ provided that deposition is not occurring, so $Q/Q_{\text{sat ice}}$ must be less than RH_{crit} or $T > 0^\circ\text{C}$. This means that extremely small ice contents produced numerically are not carried around in the model, slowing the code down. The second loop then ends.

Hence the values Q, QCL, QCF and T for each layer have been passed in and are passed back altered, and RAIN, SNOW and VT are passed in representing the fall *into* the layer and are passed back representing the fall *out of* the layer. The calculations then move down one level. When the transfers for the lowest level have been calculated the SNOW and RAIN represent surface fluxes. Q, QCL, QCF and T are the values passed out of the whole large scale precipitation scheme.

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