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VARIABLE TIMESTEP WITH DEPTH

by

K D B Whysall

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Modification Record		
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1		Missing last 2 lines of last paragraph and references (page 3) included.

NOTE: The Meteorological Office version of the Cox ocean model can be run with option U set, but with no spin up factors applied to the model integration, by using the namelist TSTEPS.

This code governs the timesteps used in the Meteorological Office version of the Cox ocean model. It reads in values for the momentum, streamfunction and tracer timesteps and replaces the single tracer timestep with an array, so enabling the tracer timestep to be defined as a function of depth. This is a strategy to accelerate the convergence of the ocean model towards equilibrium. Due to its large heat capacity, the deep ocean can take as long as 2,000 years to reach an equilibrium state. Thus it is desirable to have a method of producing an ocean model equilibrium state without using a vast amount of computer time.

By altering the timesteps used in the model we modify its timescales, effectively producing another physical system with the same equilibrium solution as the original one, but one which takes less time to reach that solution. The effect of different rates of acceleration to equilibrium should be non-existent, or at worst, negligible, when the model is at or very close to equilibrium.

Two factors are introduced into the model system of equations. These are α (the ratio between the velocity and tracer timesteps) and γ (tracer timestep length as a function of depth). If Δt_T is the tracer timestep and Δt_V, the velocity timestep, then,

$$\Delta t_T = \alpha \Delta t_V \quad (\alpha > 1)$$

The α factor reduces the phase speeds of internal gravity waves by $\alpha^{-\frac{1}{2}}$, making it possible to use relatively large tracer timesteps in order to accelerate the convergence to equilibrium of a model integration. The effect of large α on mid-latitude Rossby waves is to lower the frequency and shift the maximum frequency towards lower east-west wave-numbers (Bryan, 1984). The dispersion relation for Rossby waves of the form exp(kx+1Y-wt) becomes,

$$\omega = -\beta k[\alpha(k^2 + l^2) + \frac{f^{2]-1}}{gH_n}$$

As $(k^2 + l^2)$ tends to zero, the effect of α vanishes. Similarly, the scale of equatorially trapped waves is reduced by a factor of $\alpha^{-\frac{1}{4}}$ and the Kelvin wave speed becomes,

$$C' = (gH/\alpha)^{\frac{1}{2}}$$

The ratio of equatorial Rossby wave speed to Kelvin wave speed remains the same. These relations were all derived by Bryan (1984), assuming that γ is equal to 1 at all depths.

If the coriolis term is calculated semi-implicitly and the pressure term explicitly (as occurs in our version of the Cox model), then the stability criteria pertaining to α is,

$$\alpha > (4.0 c_1^2 \Delta t_T^2)/\Delta^2$$

(Killworth et al, 1984) where Δ is the horizontal grid spacing and c_1 the phase speed of the first baroclinic mode.

There is an order of magnitude difference between the upper and deep ocean maximum advective velocities (10^2cm/s compared to 10cm/s). Therefore it is possible to increase the tracer timestep at depth without violating the CFL criterion. Thus $\gamma(z)$ is introduced (where $d\gamma/dz < 0$) into the finite

difference temperature and salinity equations. As a model integration with steady forcing reaches equilibrium, the time derivatives will tend to zero, as will the influence of γ on the model solution. Unless the ocean is forced entirely with steady forcing, the regions
where a seasonal cycle is dominant cannot be multiplied by a γ factor. Therefore a constant tracer timestep is used over the upper ocean. However, upwelling and deep convection are seasonal phenomena which span great depths in the ocean. The effect of the γ factor on these is not clear.

It should be noted that a weighting of γ^{-1} should be included in the model's convective mixing. In the Cox ocean model, potential densities are computed at pairs of vertically adjacent tracer grid points. An unstable profile results in the tracer quantities at the two grid points being set equal to the volume average of those quantities over the two cells. This is repeated so that the stability of each cell is compared with that above and below it. As γ effectively reduces the thermal capacity of the model's lower levels, it is important to scale the level thicknesses by a factor of γ^{-1} during the above convective adjustment process.

The γ factor distorts the model stratification, with an effective Brunt-Vaisala frequency of $N^2\alpha/\gamma$, resulting in changes to equivalent depths and increasing the phase speeds of internal gravity waves. This means that, for stability, α needs to be considerably greater than would first appear, in order to compensate for this increase (Mead, 1988).

Description of the code

This code is governed by Updoc option V. If this option is selected, then the spin up factors α and γ will be applied to an ocean model integration. (They can, however, be set equal to unity using the namelist TSTEPS without recompiling the code.) The α factor is set by specifying different values for the momentum and tracer timesteps (DTUVF and DTSFF). The γ factor is set by initialising members of the array RATF (one value for each model level). DTUVF, DTSFF and RATF are all set using the namelist TSTEPS.

References

Bryan, K. (1984) 'Accelerating the convergence to equilibrium of Ocean-climate models.', JPO 666-673.

Killworth, P. D., Smith, J.M., Gill, A.E. (1984) 'Speeding up ocean circulation models.' Ocean Modelling, No. 56.

Mead, C. T. (1988) 'Asymmetries of the Oceanic Thermohaline Circulation and meridional Heat Transport.'