

**UNIFIED MODEL DOCUMENTATION PAPER NO 48**

**LATERAL BOUNDARY CONDITIONS IN THE OCEAN MODEL**

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Version 1.0

9th October 1990

Model Version

<b>Modification Record</b>		
<b>Document version</b>	<b>Author</b>	<b>Description.....</b>

Lateral boundary conditions in the ocean model  
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## Introduction

The Cox model in its basic form is written for an ocean in a closed basin; at the lateral boundaries a no-slip condition is applied and there can be no fluxes of tracers across the boundaries. A number of schemes have been devised to allow for open boundaries. Two such schemes have been incorporated into the Unified Model version of the Cox model, one devised by Adrian Gill and one by David Stevens. They have been applied to southern and northern boundaries but not to eastern or western boundaries.

This documentation provides details of these two schemes. The Gill scheme has not previously been documented in any form, and so a comprehensive description of the scheme, including the derivation of the formulae, is given here. The Stevens scheme has been described in a Met Office Internal Note (Stevens, 1988); sufficient details are therefore given here to enable code which produces the same results to be written, but reference to Stevens (1988) is necessary for background information to the development of the scheme and relevant references.

## 1. Gill scheme

### a) Introduction

This scheme may be described as of an "implicit" type - namely that the prognostic variables are not explicitly calculated for the boundary row (in comparison with the Stevens scheme, for instance), the calculation of the variables for the row adjacent to the boundary simply being modified instead. The scheme was devised for use in tropical ocean models and has only been implemented on such models at the Met. Office. It should not be seen as a rigorous method of introducing boundary conditions into the Cox model.

### b) Physics

Momentum and tracers are treated differently by the scheme and will be discussed separately.

#### i) Momentum

The primary aim in forming a boundary condition for momentum is that both inflow and outflow are permitted and in such a manner that they do not significantly affect the flow in the interior of the domain. The simplest way of doing this is to set the derivative of momentum perpendicular to the boundary to be equal to zero at the boundary:

$$\frac{\partial v}{\partial y} = 0 \quad (1.1)$$

In the tropical models, the barotropic part of the flow is set equal to zero, so there is no net mass flux across the boundaries.

#### ii) Tracers

The method of dealing with tracers needs to be more complex. The principal aims are two-fold. Firstly, tracers need to be able to be passed across the boundaries; in particular, heat needs to be able to be removed from the model domain in the case of a tropical model in order to prevent its temperatures from rising progressively. Secondly, there needs to be a smooth transition from the model-generated tracer values in the interior of the model domain to climatological values outside; this is because when the tropical ocean model is subsequently coupled to an atmospheric model which extends to higher latitudes, the ocean parameters used as a lower boundary condition for the part of the atmospheric model outside the tropical domain will need to be based on climatology, and a discontinuity in this lower boundary condition is undesirable.

The method described here satisfies these conditions. It will be discussed in terms of the temperature field, but the same principle applies to the salinity (and any other tracers which may be used).

The part of the model domain near to the boundary is considered as behaving as a type of boundary layer, in which the equation of motion may be simplified to a balance of two terms - advection and diffusion perpendicular to the boundary:

$$v \frac{\partial T}{\partial y} = A_H \frac{\partial^2 T}{\partial y^2} \quad (1.2)$$

For inflow and outflow, different solutions to Equation (1.2) are considered, as will be shown below. The case of a southern boundary will be considered, with the positive y direction northwards into the model domain.

### *I. Outflow*

For outflow ( $v < 0$ ) the following solution is found:

$$T(y) = (T_0 - T_\infty) e^{-y\sigma^{-1}} + T_\infty \quad (1.3)$$

where  $T_0$

= temperature at the boundary

$T(y)$  = temperature at distance y north of the boundary

$T_\infty$

= temperature a long way north of the boundary

$\sigma$  = width of the boundary layer =  $A_H/|v|$

Using this equation, the influence of the temperature which is applied at the boundary decays exponentially away from the boundary. In order to implement this, a boundary condition must be supplied. The smooth transition towards the climatological fields outside the domain is achieved by applying the following boundary condition:

$$\frac{\partial T}{\partial y} = \lambda (T_0 - T_c) \quad (1.4)$$

where  $T_c$  = climatological temperature at the boundary

$\lambda$  = a constant

Therefore the boundary is not fixed equal to climatology, but there is a negative feedback towards climatology; the strength of this feedback is controlled by the parameter  $\lambda$ . The value of  $T_0$  consistent with Equations (1.3) and (1.4) may be determined by combining the two equations (see Appendix 1); the resulting expression for  $T_0$  is:

$$T_0 = \frac{T_c - (1/\lambda\sigma)T_\infty}{1 - (1/\lambda\sigma)} \quad (1.5)$$

Hence if  $\lambda > \sigma^{-1}$ , then the boundary temperature  $T_0$  will tend towards the climatological temperature.

## *II. Inflow*

For inflow, the solution to Equation (1.2) is more simple, namely the trivial solution that at the boundary:

$$\frac{\partial T}{\partial y} = 0 \quad (1.6)$$

The value of the temperature at the boundary is then set equal to the climatological temperature:

$$T_0 = T_c$$

Thus water at the climatological temperature is advected into the model domain. With continued inflow, this results in a constant flux of climatological-temperature water; there is no feedback mechanism by which this inflow may be modified. The front between the incoming water and that in the model domain will be smoothed out by the diffusion in (1.2).

### c) Finite difference formulation

The finite difference formulation of the scheme is essentially straightforward. It will be described here with reference to a southern boundary, but the same principles apply to a northern boundary. Equation (1.4) is written in centred finite difference form as follows, in order to calculate the tracer values in the boundary row:

$$T_{J-1} = T_{J+1} + 2\Delta y \lambda (T_J - T_C) \quad (1.7)$$

where  $T_{J-1}$   
 = tracer at boundary row  
 $T_J$  = tracer one row in from the boundary  
 $T_{J+1}$  = tracer two rows in from boundary  
 $T_C$  = climatological value of tracer at the boundary  
 $\Delta y$  = grid-spacing perpendicular to the boundary

However, a complication was found to arise when the scheme was initially implemented, namely that the scheme developed an instability on outflow, a result of the centred finite differencing used. In order to eliminate the instability, it was found necessary to adjust the form of the finite differencing, so as to introduce additional smoothing. The value of the tracer at the central point,  $T_J$ , is replaced by a weighted mean of  $T_J$  and the average of the tracer values at the points either side ( $T_{J+1}$  and  $T_{J-1}$ ), such that:

$$T_{J-1} = T_{J+1} + 2\Delta y \lambda \left\{ \mu T_J + \frac{1}{2} (1 - \mu) (T_{J+1} + T_{J-1}) - T_C \right\} \quad (1.8)$$

The relative weighting given to  $T_J$  is a tunable parameter,  $\mu$ . The smaller the value of  $\mu$  the greater the adjustment; with  $\mu=1$ , the expression reverts to the standard finite difference form. The maximum value of  $\mu$  for which the scheme will remain stable can be determined by experiment; a value of 0.96 has been found to work and this value has been used in the Unified Model. This small amount of smoothing is sufficient to remove the instability.

#### d) Application in the Cox model

Because the scheme essentially involves adjusting the calculations for the row adjacent to the boundary in the subroutines CLINIC and TRACER for the presence of non-zero momentum and tracer fluxes into or out of the boundary row (rather than explicitly performing CLINIC and TRACER calculations for the boundary row itself), the amount of coding necessary is small.

The principal adjustments to the Cox code are in Subroutine STEP. The velocities for the boundary row are set equal to those for the row adjacent to the boundary, and the land-sea mask is treated the same way. For the calculation of the tracers, Equation (1.8) may be manipulated as follows. Multiplying (1.8) through by  $(2 \Delta y)$  and separating out the component temperature terms gives:

$$(1 - \lambda_{\Delta y} (1 - \mu)) T_{J-1} = (1 + \lambda_{\Delta y} (1 - \mu)) T_{J+1} + 2\mu_{\Delta y} T_J - 2\lambda_{\Delta y} T_C$$

Three variables, x, y & z, can be introduced to produce:

$$x T_{J-1} = y T_{J+1} + z (\mu T_J - T_C)$$

$$\text{where } x = 1 - \lambda_{\Delta y} (1 - \mu)$$

$$y = 1 + \lambda_{\Delta y} (1 - \mu)$$

$$z = 2\lambda_{\Delta y}$$

Dividing through by x:

$$T_{J-1} = \left( \frac{1}{x} \right) (y T_{J+1} + z (\mu T_J - T_C)) \quad (1.9)$$

This is the form used in the Unified Model code.

## 2. Stevens scheme

### a) Introduction

A second scheme for open boundary conditions has been written by Dr David Stevens (currently at the University of East Anglia and who spent some time in 1988 as a Vacation Consultant at the Met Office). This scheme may be described as "explicit", i.e. the prognostic variables are explicitly calculated for the boundary row (in comparison with the Gill scheme, for instance).

The scheme in its full form may be applied to northern, southern, eastern or western boundaries. It is appropriate both to ocean model configurations which include the barotropic component of the velocity (therefore requiring solution of the stream-function equation) and to those which exclude it. However, the only ocean model at the Met. Office to which the scheme has so far been applied is a model of the tropical Indian Ocean, which has only a southern boundary and excludes the barotropic component of the flow. Implementing the scheme in a different model configuration should be possible (for example, the FRAM project use this scheme), but the necessary code has not been obtained (with the exception of code for a northern boundary) or set up for the Unified Model. The code has been both set up and tested for a southern boundary and set up only for a northern boundary. This documentation therefore refers only to the open boundary scheme as set up for the baroclinic component of the flow.

Unfortunately, it has been found that, when applied to a model with non-uniform horizontal resolution in the direction of the boundary, the scheme develops a numerical instability (after several weeks of integration) in the region of the changing grid-spacing. No solution to this problem has been found (and David Stevens was also unable to offer an explanation), so implementation of the scheme to the variable-grid tropical ocean models has not, in fact, proved possible.

### b) Physics

The basis of the scheme is that it calculates the velocity and tracers on the boundary row in the model using simplified versions of the prognostic equations that are used on the interior rows.

#### i) Momentum

A linear form of the horizontal momentum equations is used:

$$\frac{\partial u}{\partial t} - fv = \frac{1}{\rho} \frac{\partial p}{\partial x} + F^u \quad (2.1)$$

$$\frac{\partial v}{\partial t} + fv = \frac{1}{\rho} \frac{\partial p}{\partial y} + F^v \quad (2.2)$$

where  $F^u, F^v$  are the diffusive terms

Thus the advective terms in the horizontal momentum equation are neglected.

## ii) Tracers

Tracers are treated differently on outflow and inflow.

### 1) Outflow

On outflow, the usual equation for the time rate of change of tracers is used except that the advection term is simplified, such that only advection perpendicular to the boundary is included (i.e. neglecting both advection parallel to the boundary and vertical advection). The basic expression is therefore:

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} = F^T \quad (2.3)$$

However, it has been shown (Stevens, 1988) that this formulation can be improved by applying a correction term to  $v$ . This is because the upstream differencing used in the advective term inhibits the propagation of internal waves out through the boundary when the fluid is being advected inwards; this can then lead to a piling up of wave energy at the boundary. The revised formulation used, therefore, when  $c$  is out of the model domain is:

$$\frac{\partial T}{\partial t} + (v+c) \frac{\partial T}{\partial y} = F^T \quad (2.4)$$

where  $c$  is the correcting phase speed. It is necessary to calculate  $c$ , and this can be done using the finite difference form of the equation:

$$\frac{\partial T}{\partial t} = -c \frac{\partial T}{\partial y} \quad (2.5)$$

at points adjacent to the boundary at the previous timestep for each level in the vertical. When the internal waves are propagating into the model domain,  $c$  is set to be zero.

## II) Inflow

On inflow, a relaxation to the climatological values of the tracers at the boundary is applied. Therefore the expression for the time rate of change of tracers in the boundary row is:

$$\frac{\partial T}{\partial t} = \alpha (T_c - T) \quad (2.6)$$

where  $T_c$   
 = climatological value of tracer on boundary  
 $\alpha$  = relaxation coefficient

A value of  $\alpha$  of 1/(30 days) has been taken, though this can be varied.

### c) Finite difference formulation

A forward differencing finite difference scheme is employed instead of the leap-frog scheme generally utilised in the Cox ocean model.

#### i) Momentum

The finite difference form of the momentum equation used for the boundary row is identical to that used for the interior rows with the following exceptions:

- i) the advective term is omitted (as per Equations (2.1) & (2.2));
- ii) the values of  $u$  &  $v$  just outside the grid needed for the calculation of the diffusive term are set equal to the values on the boundary.

#### ii) Tracers

For inflow, Equation (2.4) is represented in finite difference form as follows:

$$\frac{T_{i,1,k}^{n+1} - T_{i,1,k}^n}{\Delta t} = - \frac{(C_{i,k}^n + v_{i,1,k}^n)}{a} \frac{(T_{i,2,k}^n - T_{i,1,k}^n)}{\Delta \phi} + F_{i,1,k}^{Tn} \quad (2.7)$$

where:

$a$  = radius of the Earth

$\phi$  = angular meridional grid-point spacing at the boundary

Horizontal diffusion is omitted from the calculations, however, since it has been shown (O'Brien, 1986) that the forward differencing scheme artificially generates diffusion. Equation (2.7) may be rewritten as follows:

$$T_{i,1,k}^{n+1} = T_{i,1,k}^n + \Delta t \cdot TW_{i,1,k}^{n+1} + \left( \frac{-\Delta t}{a \cdot \Delta \phi} C_{i,k}^n \right) (T_{i,2,k}^n - T_{i,1,k}^n) \quad (2.8)$$

where

$$T_{i,1,k}^{n+1} = \frac{v_{i,1,k}^n (T_{i,2,k}^n - T_{i,1,k}^n)}{a \cdot \Delta\phi} + F_{i,1,k}^{Tn}$$

= Tracer rate of change Without correction for c

Equation (2.5) may be written in difference form:

$$\frac{-\Delta t}{\Delta y} c = \frac{\Delta t|_{y=\text{const}}}{\Delta t|_{t=\text{const}}}$$

Hence:

$$\frac{-\Delta t}{a \cdot \Delta\phi} C_{i,k}^n = \frac{T_{i,2,k}^n - T_{i,2,k}^{n-1}}{T_{i,3,k}^{n-1} - T_{i,2,k}^{n-1}} \quad (2.9)$$

The expression on the Left Hand Side can then be used in Equation (2.8). For inflow, Equation (2.6) may be written in finite difference form to give:

$$T_{i,1,k}^{n+1} = T_{i,1,k}^n + \alpha (TC_{i,1,k} - T_{i,1,k}^n) \quad (2.10)$$

#### d) Application to the Cox model

There are a few further points which need to be made, for clarity, regarding the way in which these equations are implemented in the Cox ocean model.

- i) For an open southern boundary, the "bootstrap" procedure carried out in Subroutine STEP needs to be performed for Row 1 before proceeding to the row-by-row computations.
- ii) For the calculations of baroclinic velocity, subroutine CLINIC can be called for the boundary row (Row 1 for a southern boundary, Row JMTM1 for a northern boundary) with the necessary adjustments described above. The tracer calculations are performed separately, however, and subroutine TRACER is called for the same range of rows as for a closed boundary (Rows 2 to JMTM1).
- iii) If a model integration is started from a model "dump" which was originally created without the open boundary condition, the boundary row variables will be as if the row is land. Therefore, on the first timestep the boundary row variables (including the land-sea mask) need to be read in from the row adjacent to the boundary.

Appendix 1: Gill scheme - to find the value of  $T_0$  on outflow

Equation (1.3) provides the solution to the form of the momentum equation used near the boundary and Equation (1.4) the boundary condition itself:

$$T(y) = (T_0 - T_\infty) e^{\sigma^{-1}y} + T_\infty \quad (i)$$

$$\frac{\partial T}{\partial y} = \lambda (T_0 - T_c) \quad (ii)$$

Substituting (i) in (ii) gives:

$$\frac{\partial T}{\partial y} = \frac{1}{\sigma} (T_0 - T_\infty) e^{\sigma^{-1}y} = \lambda (T_0 - T_c)$$

At the boundary,  $y=0$ . Therefore;

$$\frac{1}{\sigma} (T_0 - T_\infty) = \lambda (T_0 - T_c)$$

Rearranging:

$$T_0 \left( \frac{1}{\sigma} - \lambda \right) = \left( \frac{1}{\sigma} \right) T_\infty - \lambda T_c$$

$$\therefore T_0 = \frac{\lambda T_c - \left( \frac{1}{\sigma} \right) T_\infty}{\lambda - \left( \frac{1}{\sigma} \right)}$$

which gives Equation (1.5):

$$T_0 = \frac{T_c - \left( \frac{1}{\lambda \sigma} \right) T_\infty}{1 - \left( \frac{1}{\lambda \sigma} \right)}$$

## References

O'Brien JJ (1986) Advanced physical oceanographic modelling.  
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Stevens D (1988) Open boundaries for the Met Office version of the Cox code.