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PHYSICAL AND DYNAMIC DIAGNOSTICS AS CALCULATED WITHIN THE UNIFIED MODEL

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1. INTRODUCTION

The numerical weather prediction models give their predictions in terms of the basic variables U, V, Px, θ and q which are held on a regular lat-long grid over n ETA-levels distributed unevenly through the depth of the atmosphere. The grid and variables are specifically chosen to be favourable to the forecast model integration scheme and model output is of little use to would-be users in its raw form.

2. FIELDSFILE MODELLING

2.1 Maximum wind

The aim is to find the speed, direction and pressure of the maximum wind. First the wind components u and v are converted to speed and direction, f and d , and a search is made for the $\eta-$ level, j , with the maximum wind speed. A curve, which is a quartic in η ,is then fitted from $\eta-$ layers $j-2$ to $j+2$ in such a way that the average value, in eta, of the curve over an $\eta-$ layer is equal to the $\eta-$ level value of wind speed in that layer.

F_i are the $\eta-$ level wind speeds from the model with maximum F_i at η_i . The quartic curve, $f(\eta)$, is fitted using:

$$\int_{\eta_{k+\frac{1}{2}}}^{\eta_{k-\frac{1}{2}}} \left(f(\eta^1) - F_k\right) d\eta^1 = 0 \ , \ j-2 \le k \le j+2$$

Where $f(\eta) = a_1 + a_2\eta + a_3\eta^2 + a_4\eta^3$

Thus $$F_K = \sum_{i=1}^{n} \frac{a_i}{i} \left(\frac{\eta^i_{k-\frac{1}{2}} - \eta^i_{k+\frac{1}{2}}}{\eta_{k-\frac{1}{2}} - \eta_{k+\frac{1}{2}}} \right), \quad j-2 \le k \le j+2$$

a_i are found by inverting the matrix and then with a known the turning point (maximum wind level) is given by η_m :-

$$\frac{\delta f}{\delta \eta}\bigg|_{\eta=\eta_m} = a_2 + 2a_3\eta_m + 3a_4\eta_m = 0 \;, \; \eta_{j+\frac{1}{2}} \le \eta_m \le \eta_{j-\frac{1}{2}}$$

The maximum wind speed is found by evaluating the curve $f(\eta)$ at $\eta = \eta_m$.

The maximum wind direction is obtained by linear interpolation in log pressure between $\eta-$ levels above and below the maximum wind level.

2.2. Freezingand -20 degree levels

The freezing level, $T_0 = 0$, is located by searching upwards for the first $\eta-$ level j with $T_j < T_0$. The freezing level height and pressure are computed using:-

$$\gamma_{k=\frac{1}{2}} = \frac{T_k - T_{k+1}}{z_{k+1} - z_k} = \frac{2g\,(T_k - T_{k+1})}{c_P\,(\theta_{k+1}\,\Delta\Pi_{k+1} + \theta_k\,\Delta\Pi_k)}$$

$$H_{fz} = H_{j-1} + (T_{j-1} - T_0)/\gamma_{j-1}$$

$$P_{fz} = P_{j-1} \left(\frac{T_0}{T_{j-1}} \right)^{\frac{g}{R\gamma_{j-\frac{1}{2}}}}$$

If $T_1 < T_0$ then $H_{fz} = T_{TOP}$, $P_{fz} = P^*$

The -20 degree level height and pressure in the same manner where this time $T_0 = -20$.

2.3 Contrails

Persistent contrails are likely to be formed by aircraft at levels for which the environmental temperature is 14 degrees below the MINTRA line on a T/ϕ diagram (Helliwell and Mackenzie).The range of pressures over which persistent contrails are likely is calculated by finding the points of intersection between the environment curve and the 'MINTRA-line minus 14 degrees'.

The MINTRA-line minus 14 degrees is approximated by:-

$$T_m = a_0\, p^a 1$$

where by least square fit $a_0 = 170.1508$ and $a_1 = .046822$. The environment curve, between levels, is represented by:-

$$\frac{T(p)}{T_{j-1}} = \left(\frac{P}{P_{j-1}} \right)^{\gamma_{j-\frac{1}{2}}\frac{R}{g}} \;, \; p_j < p < p_{j-1}$$

Each $\eta-$ level j is tested for intersection having taken place in the layer η_{j-1} to η_j .

If intersection has occurred then the pressure of intersection , p_m , is found by solving simultaneously for p_m :-

$$P_m = \left\{ \frac{T_{j-1} P_{j-1}^{-\frac{R}{g}\gamma_{j-\frac{1}{2}}}}{a_0} \right\}^{\frac{1}{a_1 - \frac{R}{g}\gamma_{j-\frac{1}{2}}}}$$

Having found all the points of intersection, if any, the highest and lowest pressures are converted to ICAO heights in thousands of feet.

2.4 Humidity.

The forecast model produces values of specific humidity, q , on $\eta-$ surfaces, whereas most users require Relative Humidity on pressure surfaces.

The first stage in this calculation is the evaluation of the saturation mixing ratio, q_{SAT} , on $\eta-$ levels. A lookup-table is used to find the saturation vapour pressure, e_s from the temperature, T . The lookup table used is identical to the one in the forecast model; it assumes saturation with respect to water for $T > -5°$, and with respect to ice for $T < -10°C$.

For temperatures between $-10°C$ and $-5°C$, the value given is an interpolation between the value obtained by assuming saturation with respect to water and the value obtained by assuming saturation with respect to ice. Then the saturation mixing ratio on a $\eta-$ level is:-

$$q_{SAT} = \frac{.622 e_s}{P}$$

The relative humidity is then calculated from

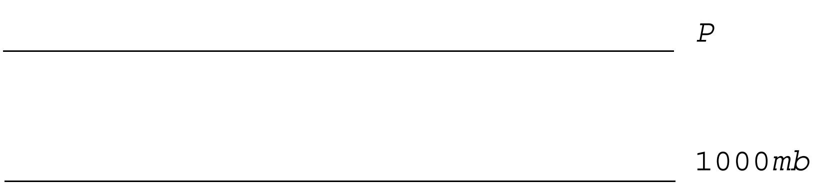
$$RH = \frac{q}{q_{SAT}} \; x \; 100$$

and interpolated linearly in log pressure to pressure surfaces. If the pressure surface is below η_1 then the relative humidity at η_1 is specified.

2.5 Wet-bulb potential temperature

This calculation is done in two stages. First the isobaric wet-bulb temperature is found, then the wet-bulb potential temperature is calculated by increasing the pressure to 1000 mb whilst maintaining saturation, (descending a saturated adiabat line on a tephigram, Fig 1).

Fig 1.



(i) Calculation of isobaric wet-bulb temperature

This is defined as the temperature to which a mass of air can be cooled by adding water at constant pressure and temperature T_W , where T_W is the wet-bulb temperature. Then

$$(T_a - T_W)(C_p + r_a C_{pV}) = (r_S(T_W) - r_a)L(T_W) \quad (1)$$

where

T_a =	initial temperature of the air
r_a =	initial mixing ratio of the air
C_p =	specific heat capacity of dry air
C_{pV} =	specific heat capacity of water vapour
$r_s(T_W)$ =	saturation mixing ratio at temperature T_W
$L(T_W)$ =	latent heat of evaporation at temperature T_W

(ii) Calculation of wet-bulb potential temperature

The equation for descent of a saturated adiabatic is obtained by relating temperature to pressure as pressure is increased whilst saturation is maintained (this implies that moisture is being added). Using the first law of thermodynamics,

$$-Ldr_s = C_p dT - \frac{RT}{m_d}\frac{d}{p}$$

(See below for definition of new variables)

The change required in r_s can be related to the change in temperature at a given pressure by using the Clausius Clapyron equation;

$$\frac{de_s}{e_s} = \frac{m_v L}{R}\frac{dT}{T^2}$$

and the definition of r_s

$$r_s = \frac{\varepsilon e_s}{p-e_s} \cong \frac{\varepsilon e_s}{p} \quad Nb \quad q_s = \frac{\varepsilon e_s}{p-e_s(1-\varepsilon)}$$

Then

$$dr_s \cong \varepsilon d\left(\frac{\varepsilon e_s}{p}\right) = \frac{\varepsilon e_s}{p}\left(\frac{de_s}{e_s} - \frac{dp}{p}\right)$$

$$= \frac{\varepsilon e_s}{p}\left(\frac{m_v L}{R}\frac{dT}{T^2} - \frac{dp}{p}\right)$$

Substituting in to the expression above for r_s gives

$$\Delta T = \left(\frac{Lq_s + \frac{RT}{md}}{\left(C_p + \frac{m_v L^2 q_s}{RT^2}\right)p}\right)\Delta p$$

where p = pressure

$e_s = SVP\ of\ water \cong 6.11 \exp\left(\frac{m_V L}{R}\left(\frac{1}{273} - \frac{1}{T}\right)\right)$ from Clausius-Clapyron

R = universal gas constant

ε = .622

m_V = molecular weight of water vapour

m_d = molecular weight of dry air

Numerical Calculation

(iii) Calculation of T_W

Rearranging (1), and approximating $L(T_W)$ by $L(T_a)$ gives

$$r_S(T_W)L(T_a) + T_W(C_p + r_a C_{pv}) = r_a L(T_a) + T_a(C_p + r_a C_{pv}) \quad (3)$$

The right hand side is then known.

Defining $g(T) = r_s(T)L(T_a) + T_a(C_p + r_a C_{pv})$,then $g(T_W)$ is known (4)

$$g'(T) = \frac{\varepsilon m_v L^2(T_a)e_s}{pRT^2}(T) + C_p + r_a C_{pv} \text{ for isobaric process} \quad (5)$$

T_W can then be found iteratively beginning with first guess T_a

$(i+1)$ st guess is

$$T_{i+1} = T_i + \frac{g(T_W) - g(T_i)}{g'(T_i)} \quad (6)$$

(Newton-Raphson method)

This is continued until $|g(T_{i+1}) - g(T_W)| < \delta$ where $\delta = 1$.

(iv) Descent of saturated adiabat

Equation (2) may be written as

$$dp = f(T,p)dT$$

T can then be incremented and corresponding pressure increments calculated. In order to take account of the non-linearity of this equation, f is first calculated at a point $(T_i + \Delta T, p_i + \Delta' p_i)$ where $\Delta' p_i = f(T,p)\Delta T$

(ΔT is chosen to be small enough that decreasing it further has no significant effect on the result).

The gradient f is then averaged between the two pressures p_i and $p_i + \Delta' p_i$ so that

$$\Delta p = \frac{1}{2}\left(f(T_i + \Delta T,\ p_i + \Delta' p_i) + f(T_i, p_i)\right)\Delta T$$

and $$p_{i+1} = p_1 + \Delta p_i \quad , \quad T_{i+1} = T_i + \Delta T$$

This is repeated until p is greater than 1000 mb. The temperature at 1000mb is then linearly interpolated between values at either side so that

$$\theta_W = T_{(1000)} = \frac{T_j(p_{j+1} - 1000) + T_{j+1}(1000 - p_j)}{p_{j+1} - p_j} \quad where \quad p_j < 1000mb, \quad P_{j+1} \geq 1000mb$$

2.6. Clear air turbulence (CAT)

To assist the aviation forecasters, charts of probability of CAT for 300, 250 and 200 mb are produced. An empirical formula, derived by Dutton (1979) is used to compute a CAT predictor index from the model forecast fields:-

$$E = 1.25 S_H + 0.25 S_V^2 + 10.5$$

where S_H is the horizontal wind shear in ms^{-1} per 100 km

$$S_H = \left\{ uv\frac{du}{dx} - u^2\frac{du}{dy} + v^2\frac{dv}{dx} - uv\frac{dv}{dy} \right\}(u^2 + v^2)^{-1}$$

where x and y are co-ordinates on a Cartesian Rectilinear map projection and S_V is the vertical wind shear in ms^{-1} per km.

$$S_V = \left\{ \left(\frac{du}{dp}\right)^2 + \left(\frac{dv}{dp}\right)^2 \right\}^{\frac{1}{2}} \frac{dp}{dh}$$

The vertical gradients $\frac{du}{dp}$ and $\frac{dv}{dp}$ are found by fitting and evaluating cubic splines to the u and v wind components on the standard pressure levels.

The value of $\frac{dp}{dh}$ used is determined from the ICAO standard atmosphere.

The index E is then converted to a probability per 100 km of flight length, P, by interpolating linearly between the following empirical values:-

E	**P**
5.0	0.0
7.5	0.95
10.0	1.35
15.0	1.85
20.0	2.05
28.5	3.30
32.0	4.75
35.5	7.50

2.7 Snow probability

Snow probability is calculated according to the formula

$$P = 40(130.5 - (h_{850} - .96667h_{1000}))$$

and $0 \le p \le 1000$

where h_{850} and h_{1000} are 850 mb and 1000 mb heights.

P is then the probability expressed as a percentage.